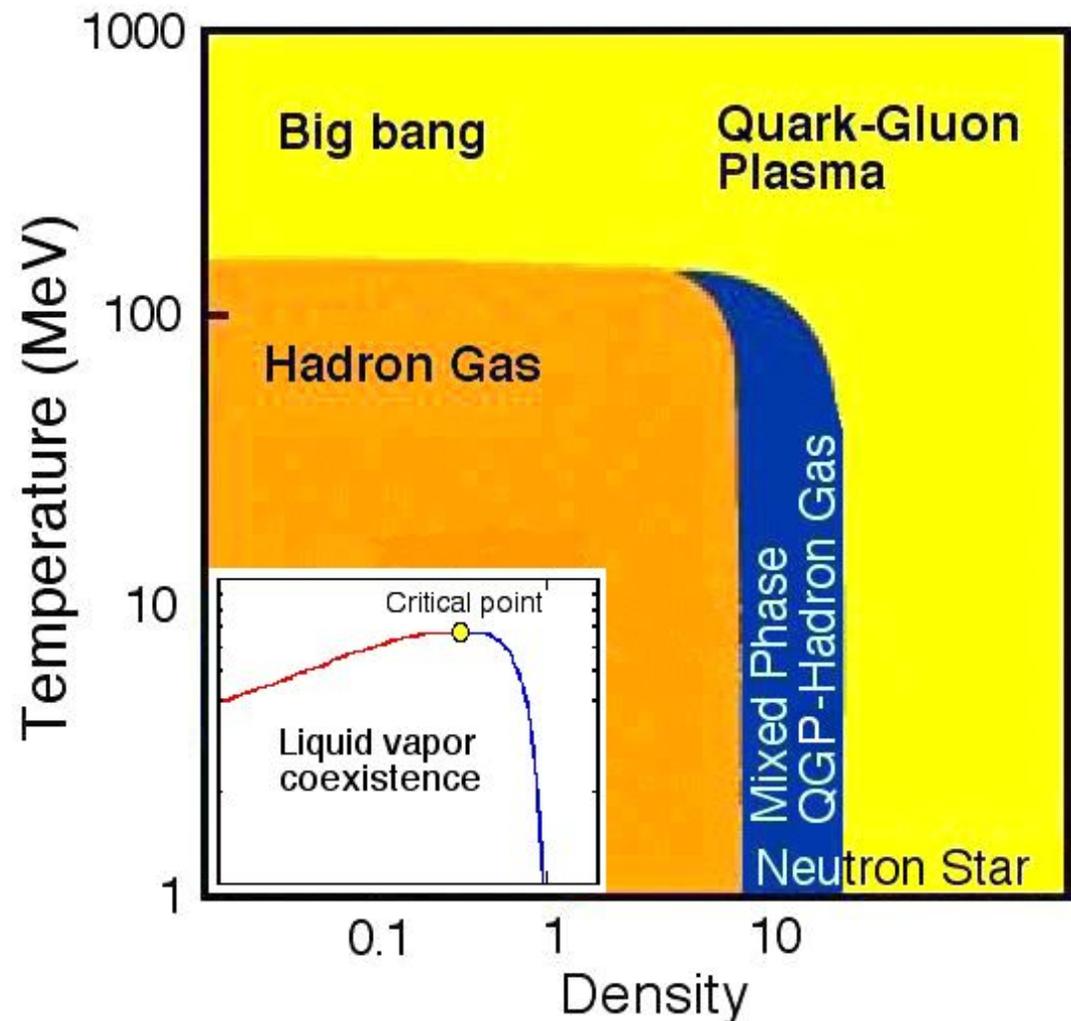


Lessons Learned: Liquid-Vapor Nuclear Matter Phase Diagram

*Larry Phair, P.T. Lake, L. G. Moretto
Nuclear Science Division, Lawrence Berkeley National Lab*

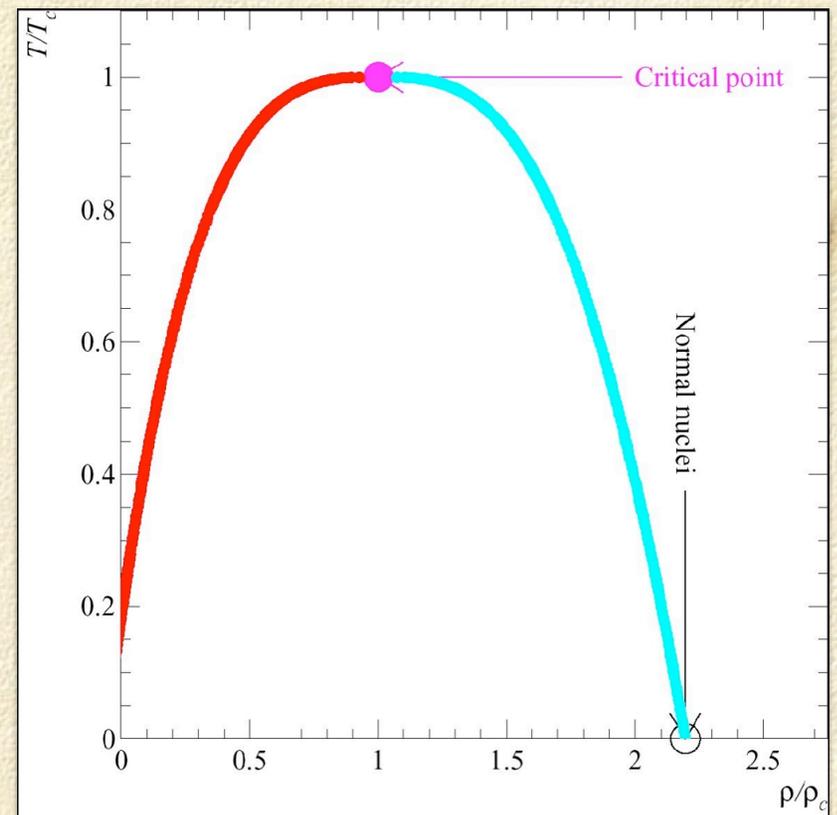
History & why you should care

- 80's and 90's: characterization of the liquid vapor phase transition
- multifragmentation: "new" process
- Lessons learned in producing the phase diagram



Why there are so few nuclear phase diagrams...

- The liquid vapor phase diagram – 3 problems:
 1. Finite size: How to scale to the infinite system?
 2. Coulomb: Long range force
 3. No vapor in equilibrium with a liquid drop. Emission into the vacuum.



Ideal gas law and cluster picture

Non-ideal gas of monomers, write it in terms of partial pressures of the clusters (of size A)

□ Pressure:
$$p(T) = \sum_A p_A(T) = T \sum_A n_A(T),$$

□ Density:
$$\rho = \sum_A A n_A(T),$$

Cluster concentrations (n_A) are everything

Fisher Droplet Model (FDM)

- FDM developed to describe formation of drops in macroscopic fluid
- The FDM allows us to approximate a real gas with an ideal gas of clusters (monomers, dimers, trimers, etc.)
- The FDM provides a general formula to describe the concentrations $n_A(T)$



Lesson learned: T_c and c_0 appear together

$$\begin{aligned}
 \square \quad n_A(T) &= \boxed{g(A)} \exp\left(-\frac{c_0 A^\sigma}{T}\right) = q_0 A^{-\tau} \exp\left(c_0 A^\sigma \left[\frac{1}{T_c} - \frac{1}{T}\right]\right) \\
 &\quad \downarrow \\
 & q_0 A^{-\tau} \exp(\omega A^\sigma) \quad \omega = \frac{c_0}{T_c} \quad \text{surface free energy}
 \end{aligned}$$

I. Finite size effects: Complement

Moretto *et al.* PRL **94**, 202701 (2005)

Infinite liquid A_0
Ocean Admits the **same** T_c as the **infinite** system, and c_0

$$n_A(T) = g(A) \exp\left(-\frac{E_S(A)}{T}\right) = q_0 A^{-\tau} \exp\left(-c_0 A^\sigma \left(\frac{1}{T} - \frac{1}{T_c}\right)\right)$$

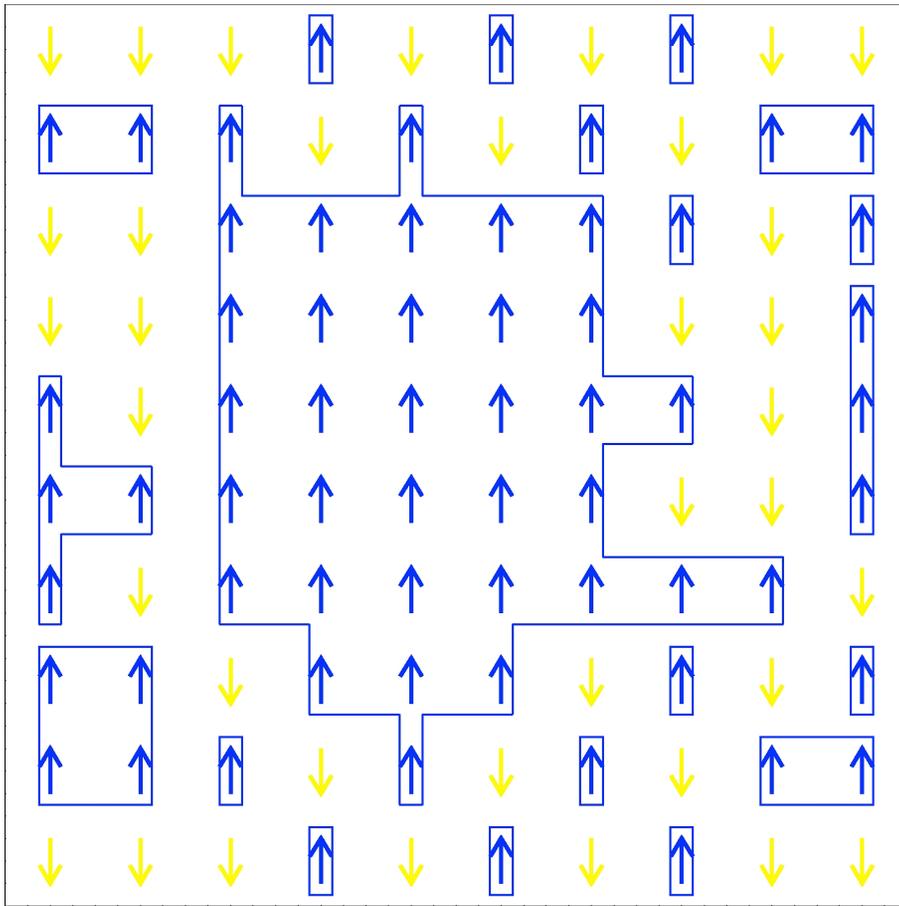
Finite liquid drop A_0 \Rightarrow $A_0 - A$ $\bullet A$

$$n_A(T, A_0) = \frac{g(A)g(A_0 - A)}{g(A_0)} \exp\left(-\frac{E_S(A) + E_S(A_0 - A) - E_S(A_0)}{T}\right)$$

Fisher+Complement

$$n_A(T, A_0) = q_0 \frac{A^{-\tau}(A_0 - A)^{-\tau}}{A_0^{-\tau}} \exp\left(-c_0 [A^\sigma + (A_0 - A)^\sigma - A_0^\sigma] \left(\frac{1}{T} - \frac{1}{T_c}\right)\right)$$

Ising model (or lattice gas)



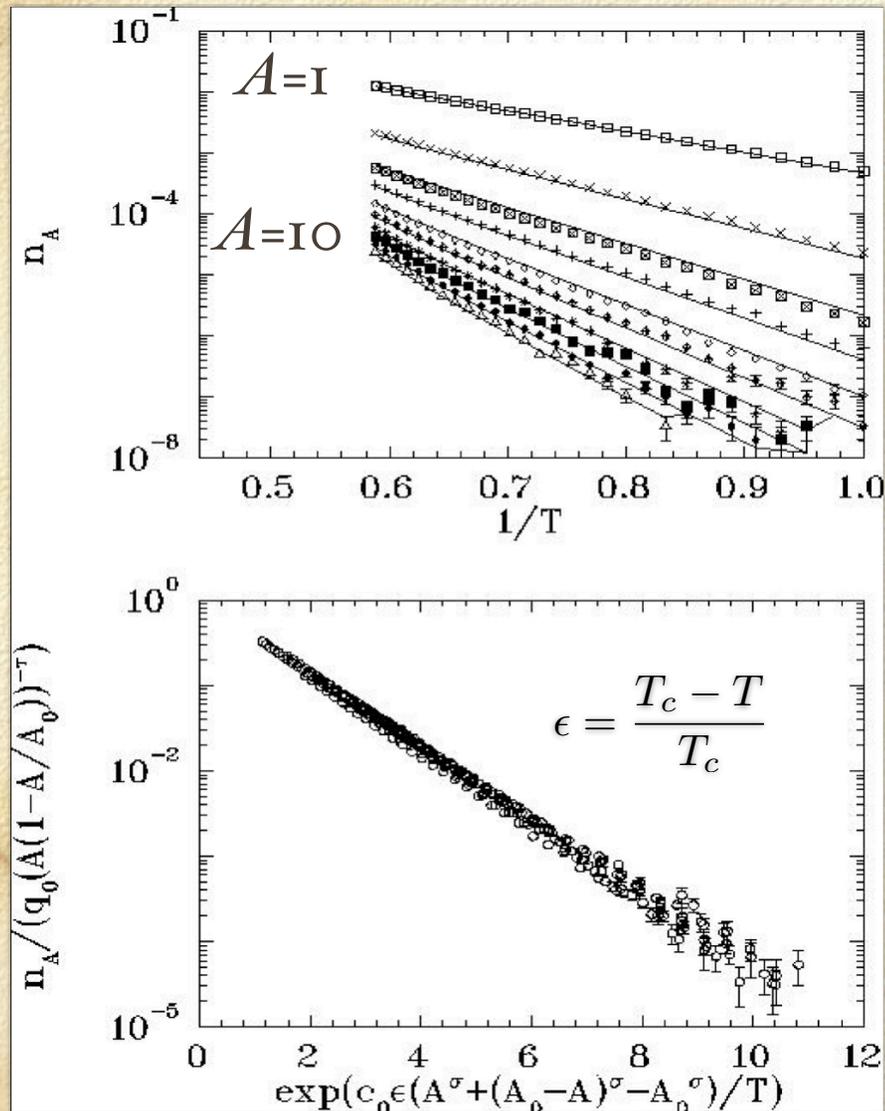
- Magnetic transition
- Isomorphous with liquid-vapor transition
- Hamiltonian for s -sites and B -external field

$$H = - \sum_{\langle ij \rangle} J_{ij} s_i s_j - B \sum_{i=1}^N s_i$$

$$s = \begin{cases} 1 \\ -1 \end{cases} \Rightarrow \frac{s+1}{2} = \begin{cases} 1, \text{ occupied} \\ 0, \text{ empty} \end{cases}$$

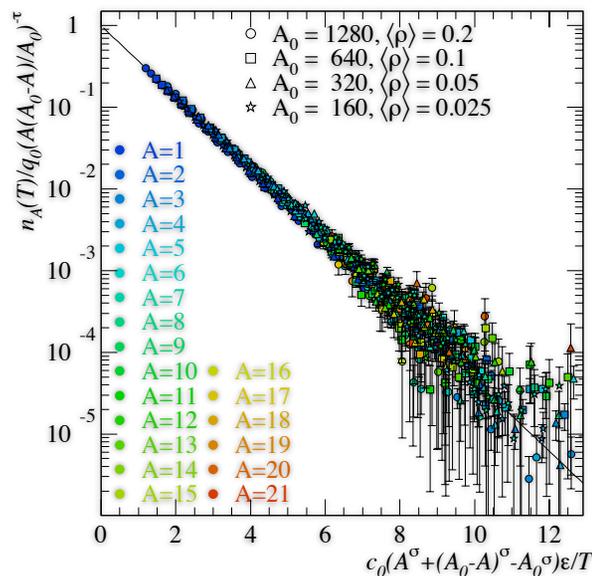
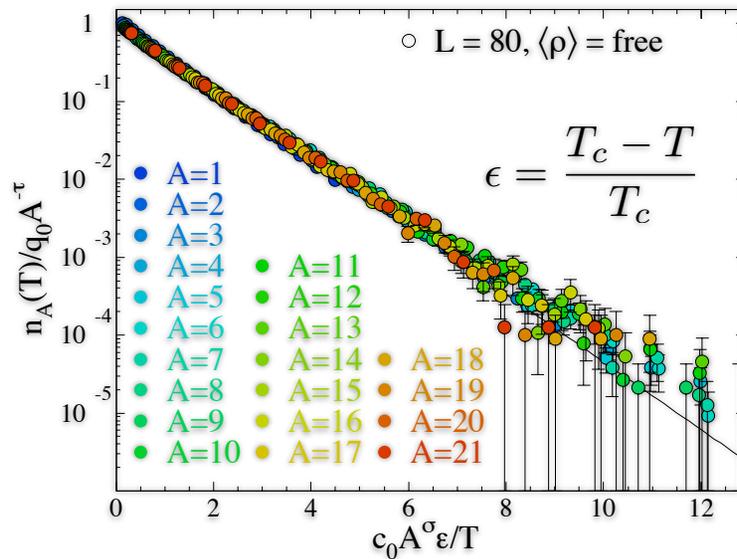
$$J_{ij} = \begin{cases} J, & i \text{ and } j \text{ neighboring sites} \\ 0, & \text{otherwise} \end{cases}$$

Test Complement with Ising model



- 2d lattice, $L=40$, $\rho=0.05$, ground state drop $A_0=80$
- Regular Fisher, $T_c=2.07$
- Taking into account the finite system $T_c = 2.32 \pm 0.02$ to be compared with the theoretical value of 2.27...
- Can we declare victory?

Scale many systems (Ising droplets)



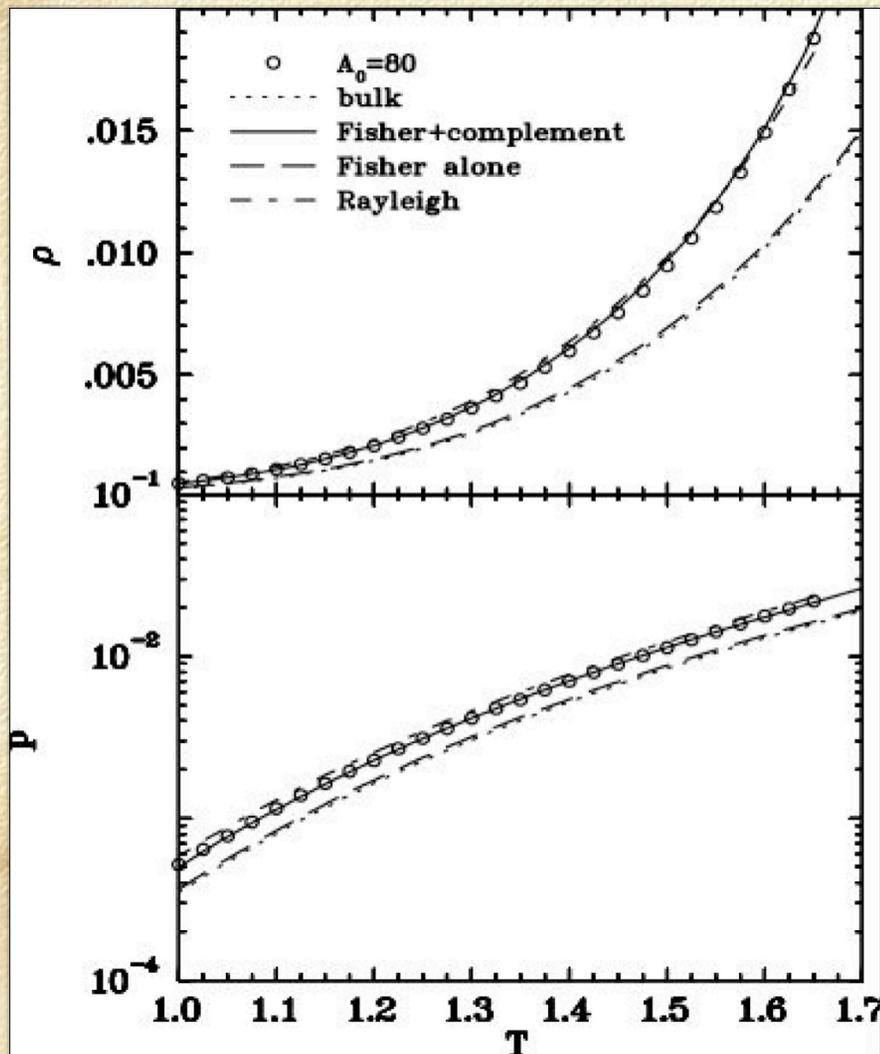
Lesson learned: finite systems – coexistence ends long before T_c

□ $T_c = 2.29 \pm 0.01$ for the free magnetization case to be compared with the theoretical value of 2.27...

□ $T_c = 2.30 \pm 0.01$ for the fixed magnetization

Lesson learned: with the right physical picture, **all** of the data point to T_c

From finite pressure and density to bulk values



■ Onsager solution for the magnetization mapped to density (dotted line)

■ Fisher+complement

■ Let $A_0 \rightarrow \infty$ (dashed line), recover Onsager!

$$\lim_{A_0 \rightarrow \infty} n_A(A_0, T) = n_A(T)$$

Lesson learned: T_c is a property
of the infinite system

Problem 1: Finite size

Fisher+complement $\lim_{A_0 \rightarrow \infty} n_A(A_0, T) = n_A(T)$ Fisher

- How to scale to the infinite system? Solved.
Put a liquid drop “extension” (complement)
into the Fisher formula.
- At the same level of approximation as the
original Fisher expression
- Start with a finite system, recover bulk T_c, c_0 ,
density and pressure values
- And vice versa, start with infinite system and
go to finite system

Problem 2: Coulomb – definition of phases (?)

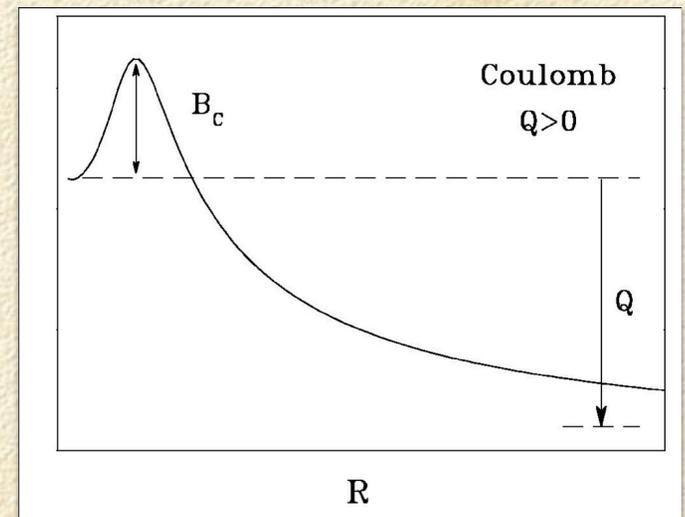
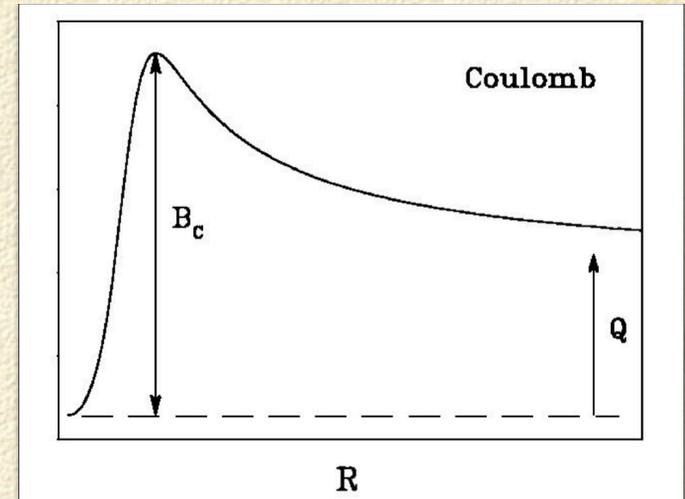
- Phases are uniform homogeneous systems
- Phase transitions \Rightarrow short range interactions
- Long range interactions: difficult to define phases

What to do with the Coulomb energy?

- $E_C = E_{DSE} + E_{DV} + E_{VSE}$
- E_{DSE} : Drop self energy (EASY)
- E_{DV} : Drop-vapor interaction energy. (Take the vapor at infinity!!)
- E_{VSE} : Vapor self energy. (Diverges for an infinite amount of vapor!!)

The problem of the droplet-vapor interaction

- If each cluster is bound ($Q < 0$), OK.
- If at least one cluster seriously unbound ($|Q| \gg T$), then trouble.
Entropy problem. $\Delta F = \Delta E - T\Delta S$
For a dilute phase at infinity, this spells disaster!
At infinity,
 ΔE is very negative
 ΔS is very positive
 ΔF can never become zero.



Vapor self energy

- Infinite vapor, self energy diverges (with Coulomb)
- Try taking a small sample of a dilute vapor so that $E_{VSE}/A \ll T$
- Alternatively, we could consider a finite box containing a finite system. Unfortunately, at any other distance smaller than infinity the result depends annoyingly on the size (and shape) of the container!
- inelegant and non-general situation leading to confusing questions about true equilibrium.

Use a box? Results will depend on size (and shape!) of box.

- **A box provided by nature is the only way out!**

Solution: make your own box

- Van der Waals liquid, ΔH_m increases with increasing A and saturates for infinite systems. Add Coulomb. Put it in a box.

$$E_b \approx a_v A + a_s A^{2/3} + a_c Z^2 \left(\frac{1}{A^{1/3}} - \frac{1}{nA^{1/3}} \right)$$

- ΔH_m begins to decrease when $\partial\mu/\partial A=0$,

$$(\mu = \Delta H_m = \partial E_b / \partial A)$$

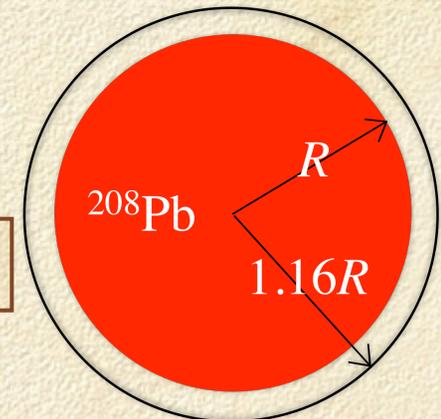
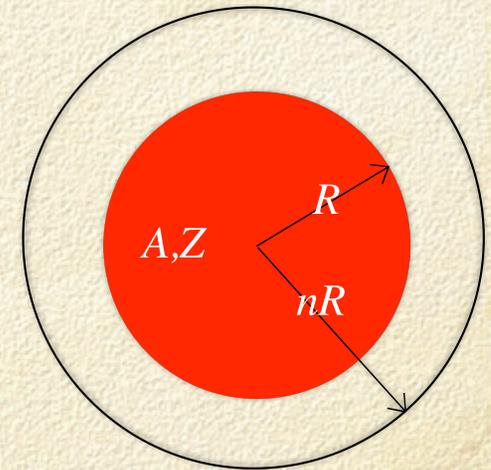
$$\hat{A} = \frac{1}{5} \frac{a_s}{a_c} \frac{1}{K^2} \frac{n}{n-1}, \quad K = \frac{Z}{A}$$

- Without a box, or as $n \rightarrow \infty$, $A=29$ is where ΔH_m begins to decrease

$$V = 1.56 V(^{208}\text{Pb})$$

- For ^{208}Pb in a box, $\partial\mu/\partial A=0$ when $n=1.16$

- Coulomb problem is “solved” ($\Delta F=0$ is possible), but no room for vapor

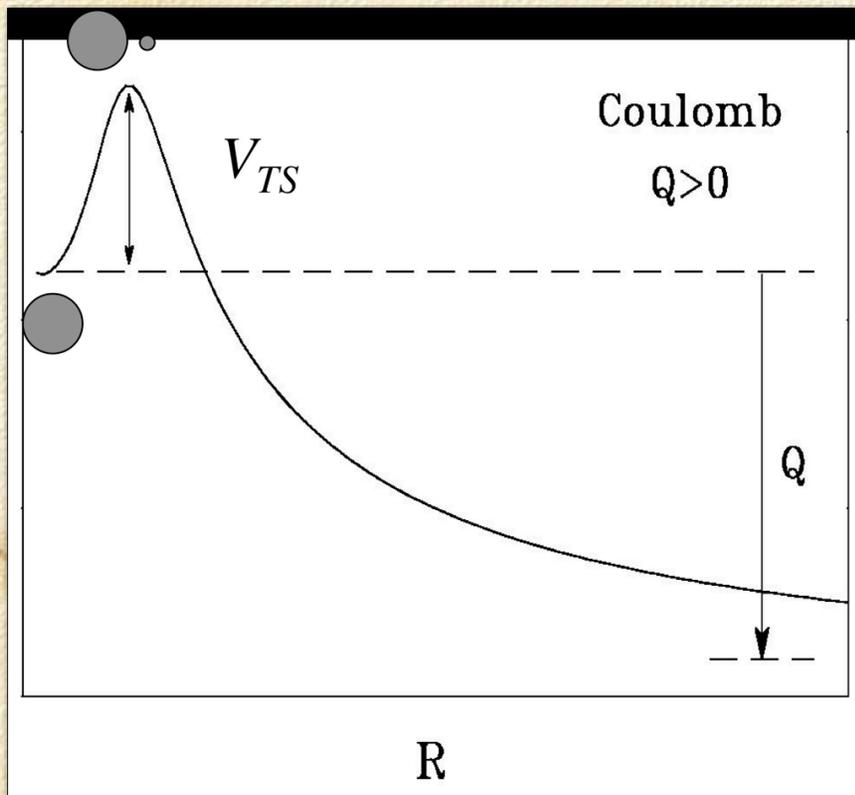


(to scale)

Lesson learned:
We must define
the phases if we
are going to
discuss phase
transitions.

How to deal with Coulomb?

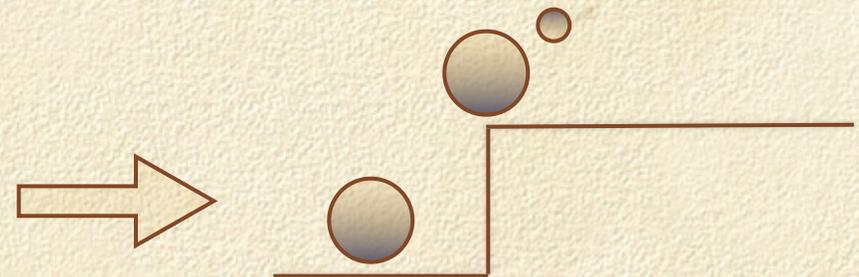
Transition state



Van der Waals
concentration

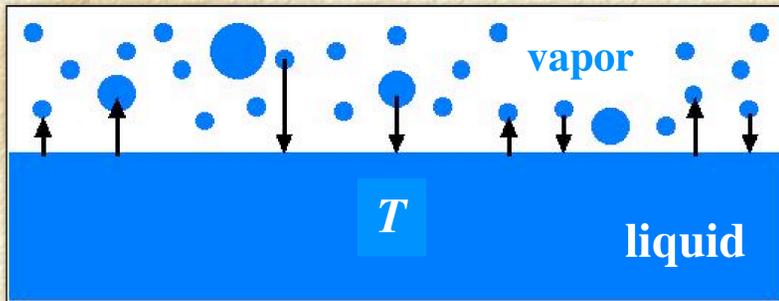
$$n = n_0 e^{-\frac{V_{TS}}{T}} = \boxed{n_0 e^{-\frac{V_{Surf}}{T}}} e^{-\frac{V_C}{T}}$$

n_{VdW}



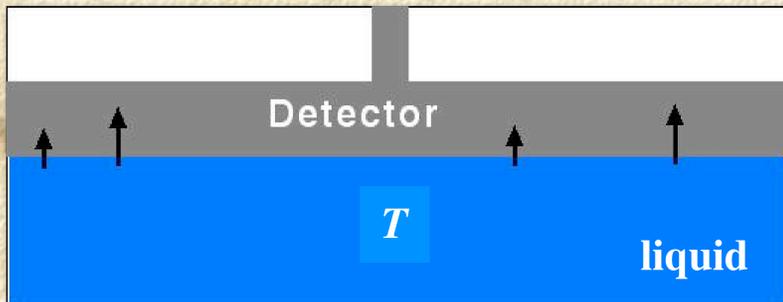
Problem 3: no physical vapor in equilibrium

Equilibrium bulk liquid in coexistence with its vapor



- Is there a gas phase in equilibrium with the droplet? (NO)

Equilibrium bulk liquid evaporating into a detector



- Can we still make a thermodynamic characterization of the gas phase? (YES)

$$\frac{\Gamma}{\hbar} = n_A(T) \langle v_A(T) \sigma(v_A) \rangle$$

ISiS measured lifetimes compared to yields

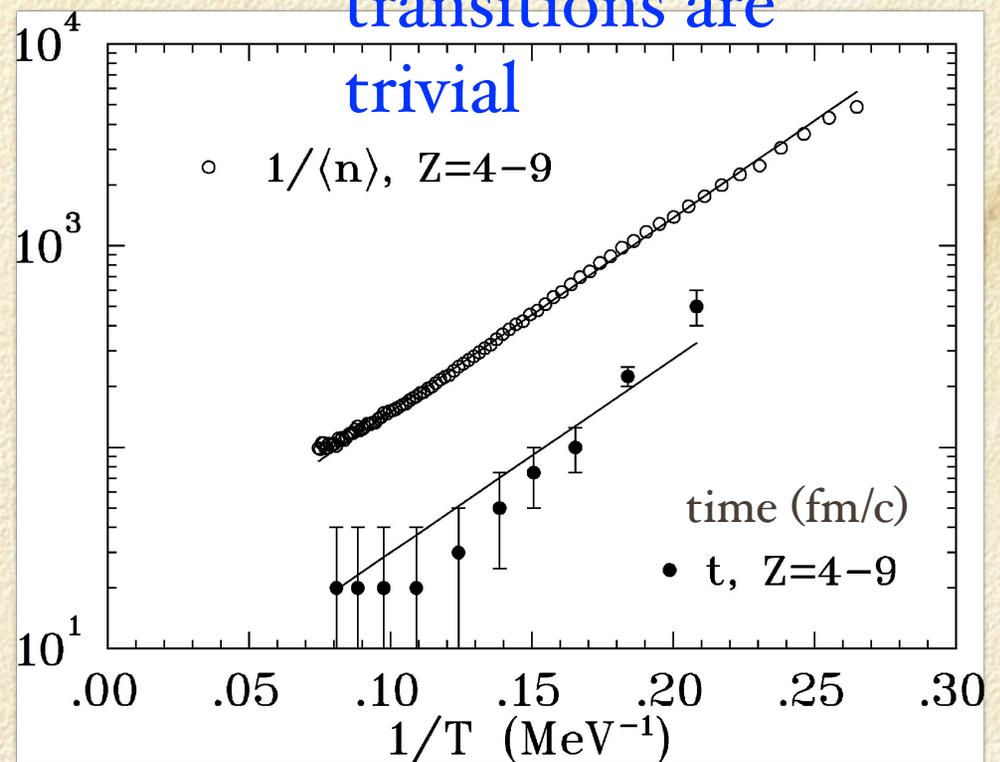
Lesson learned:
First order phase
transitions are
trivial

- Experimentally measured lifetimes and yields both controlled by the **same** Boltzmann factor

$$\langle n \rangle \propto \Gamma \propto e^{-B/T}$$

$$t = \frac{\hbar}{\Gamma} \propto e^{B/T}.$$

- Evidence for a rate description of the physical process

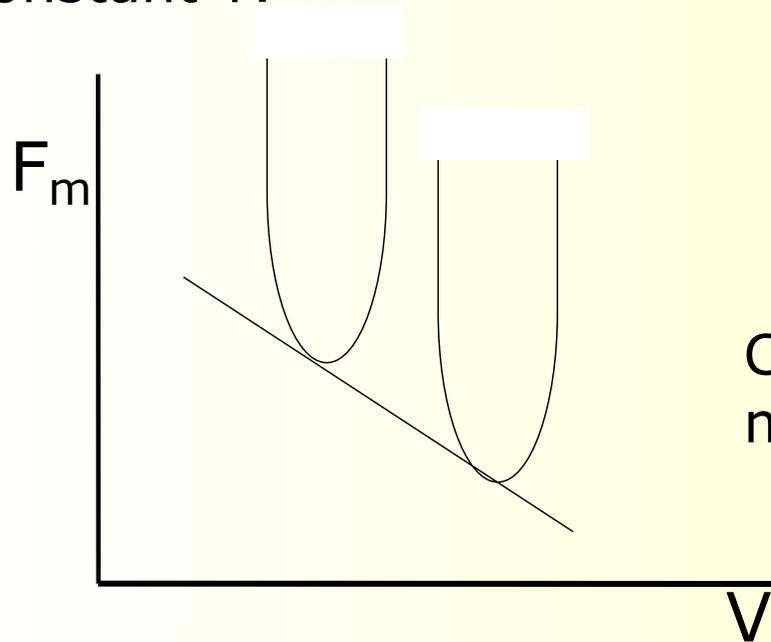


Lifetimes: L. Beaulieu et al., Phys. Rev. Lett. **84**, 5971 (2001) & PRC **63**, 031302 (2002)

Yield comparison: L. G. Moretto et al., arXiv/nucl-ex/0209009, LBNL-51306

Consider two different phases like two condensed phases with different packing (e.g f.c.c. and b.c.c.).

Calculate for each the molar free energy as a function of V at constant T .



$$\left. \frac{\partial F}{\partial V} \right|_T = -p$$

Common tangent
minimizes free energy

Conclusion: 1st order phase transitions are describable in terms of the thermodynamic properties of each phase

considered independently.

Equilibrium conditions do not require the contact of the two phases!

Solutions found

- Going from the finite system to the infinite system and vice versa: Complement

$$T = \sqrt{\frac{E^*}{A} 8 \left(1 + \frac{E^*}{E_{LD}(Z_0, A_0)} \right)}$$

$$A = 2Z \left(1 + \frac{E^*}{E_{LD}(Z, 2Z)} \right)$$

- Coulomb and defining phases: Transition state

$$E_i = E_{LD}(Z_0, A_0) + E_{rot}(A_0, J)$$

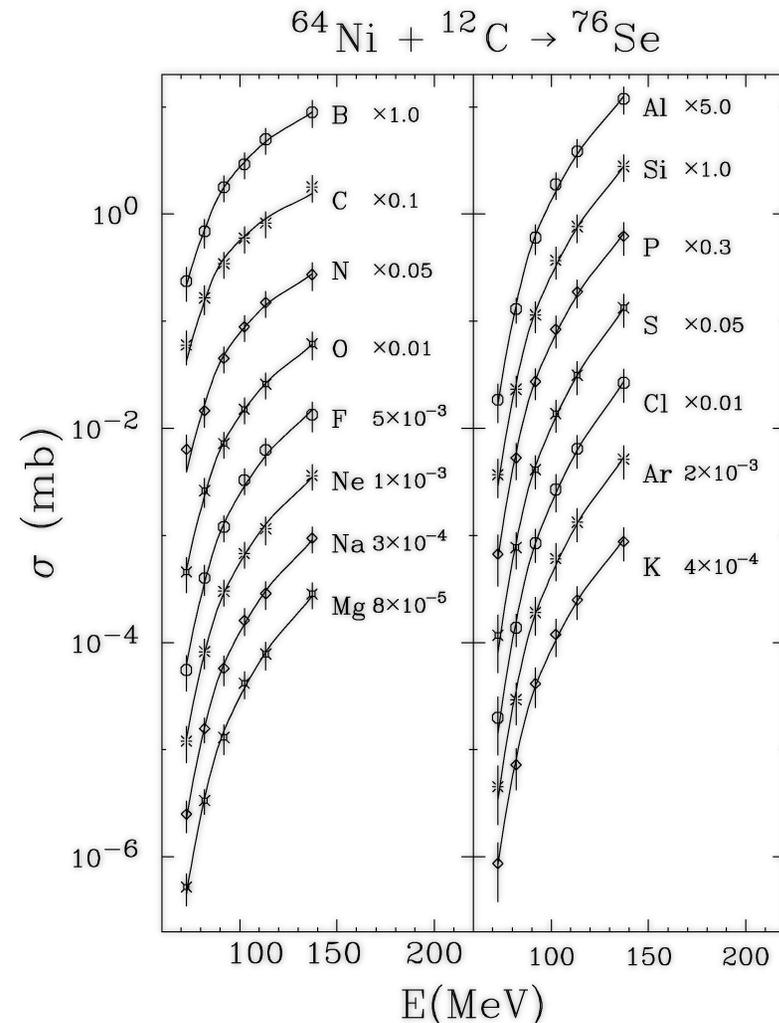
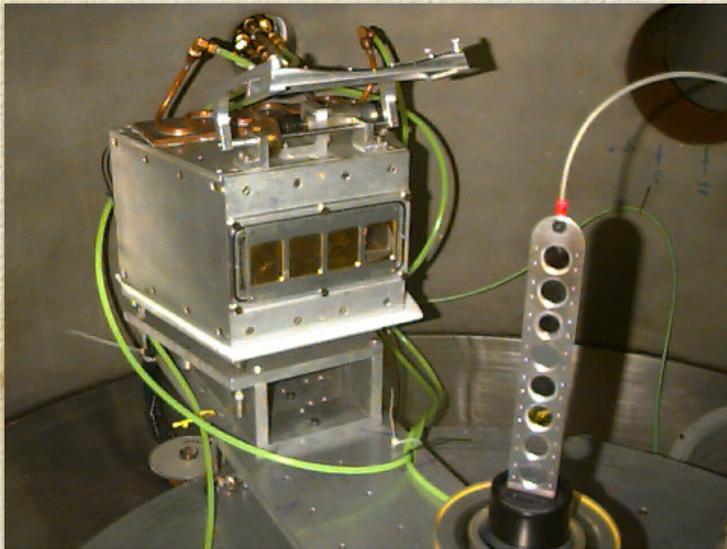
$$E_f = E_{LD}(Z, A) + E_{LD}(Z_0 - Z, A_0 - A) + E_{rot}^f + \frac{Z(Z_0 - Z)e^2}{r_0(A^{1/3} + (A_0 - A)^{1/3}) + d}$$

- No vapor: Use the rates (with Coulomb removed) and infer the concentrations of a “virtual” vapor

$$J = J_0 + J_1 E^*$$

Compound nucleus data (88-Inch Cyclotron)

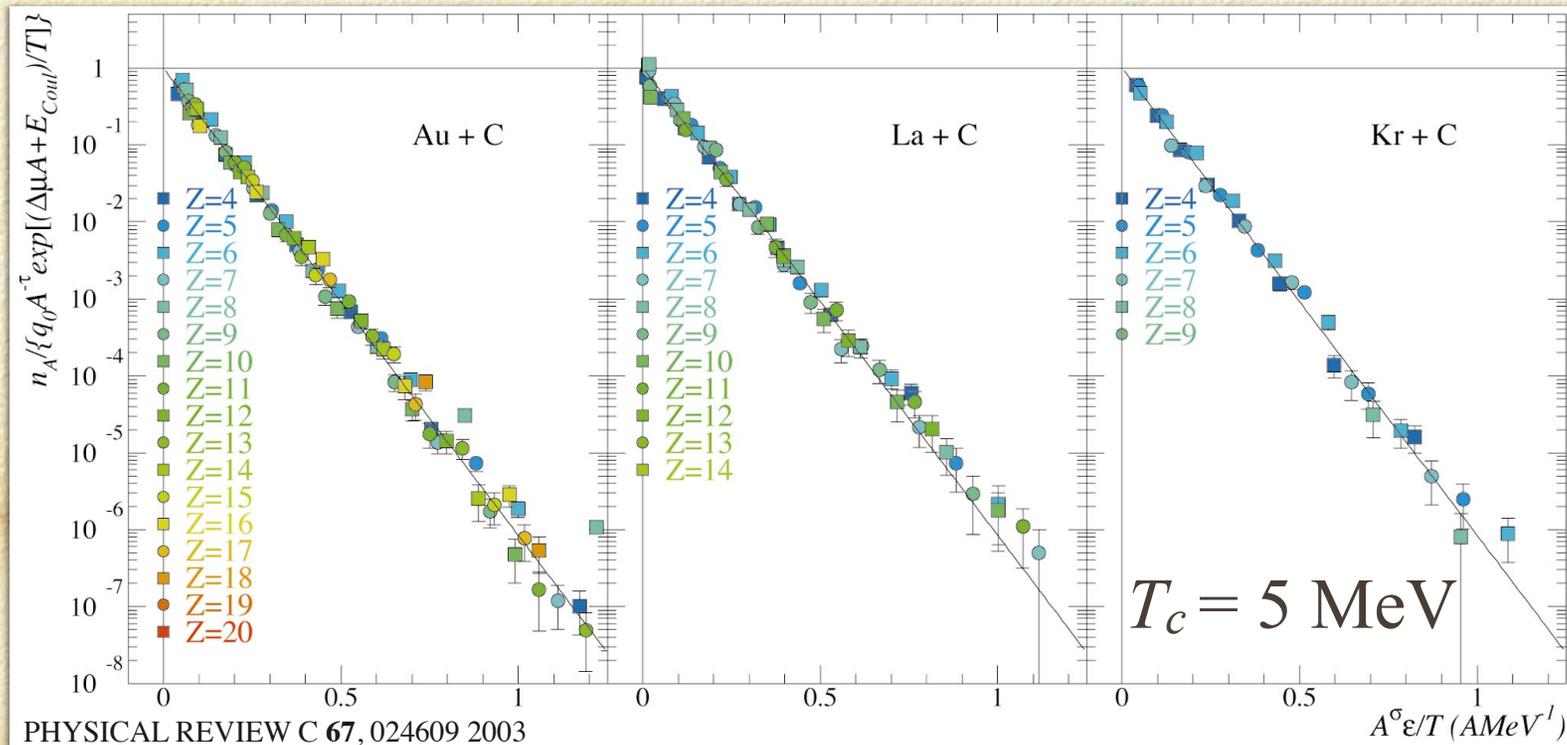
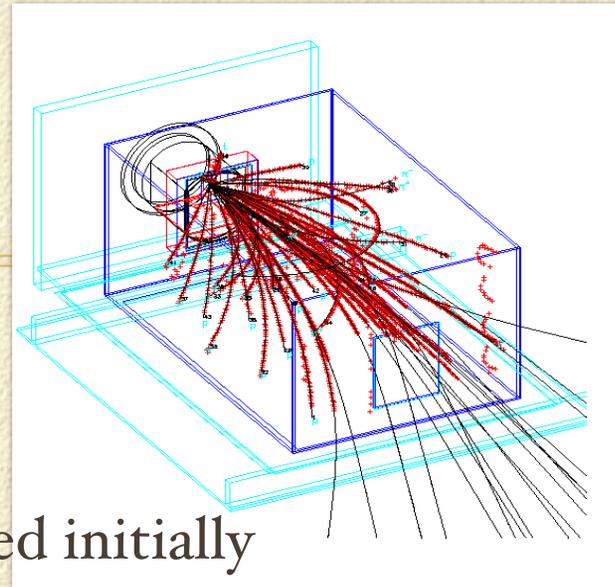
- Old school: gas Si telescopes
- Reverse kinematics reaction



EOS TPC data (Bevelac)

□ E/A= 1 GeV

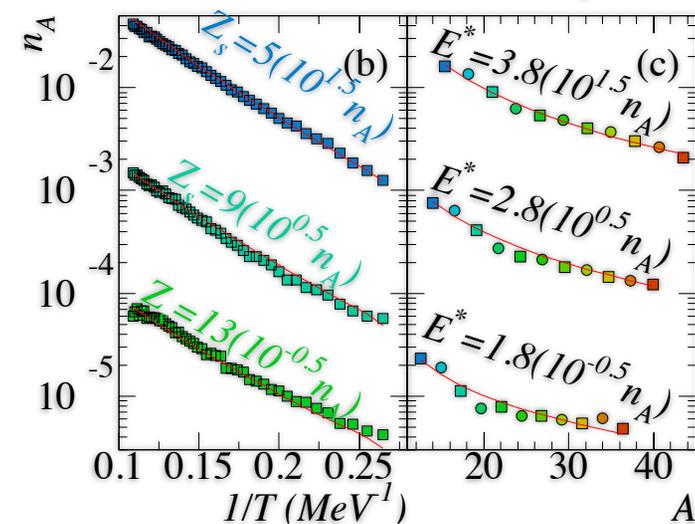
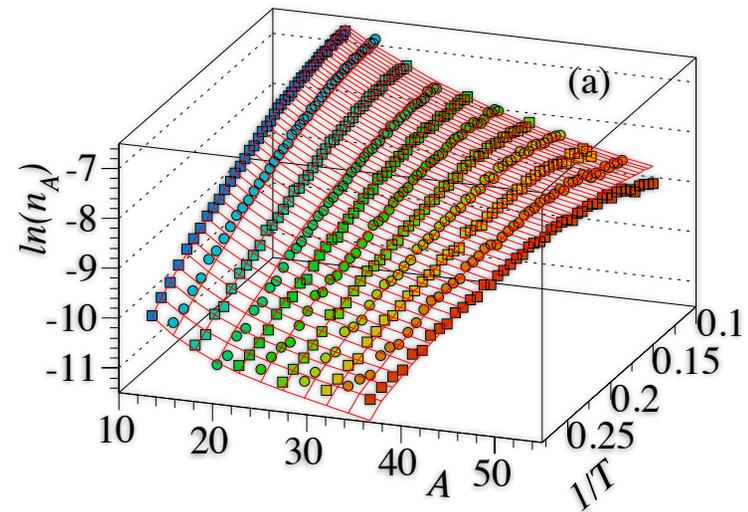
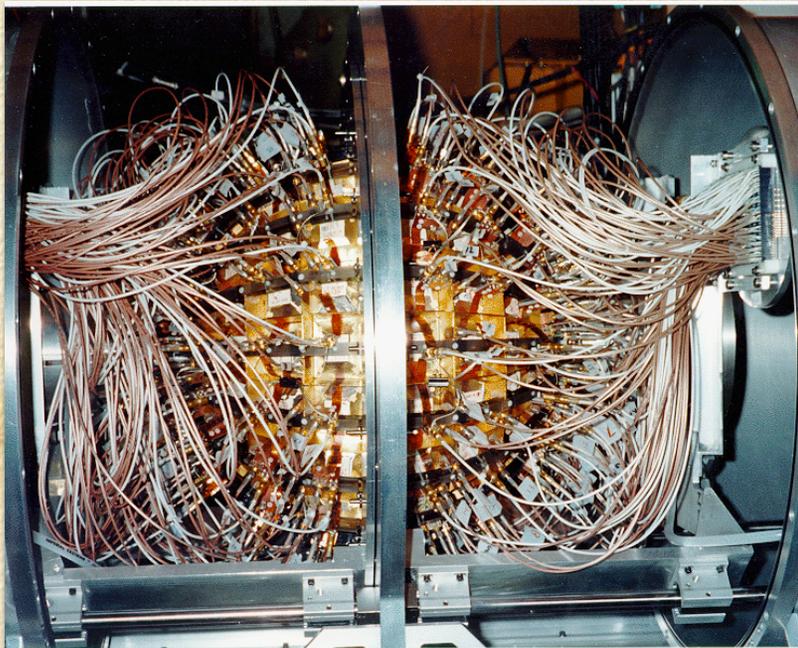
□ Finite size effects not appreciated initially



ISiS data (AGS)

□ Gas-Si-CsI sphere

□ $\pi + \text{Au}$, $E = 8 \text{ GeV}$



All data

$\pi + {}^{197}\text{Au}$, 8 GeV

${}^{84}\text{Kr} + {}^{12}\text{C}$

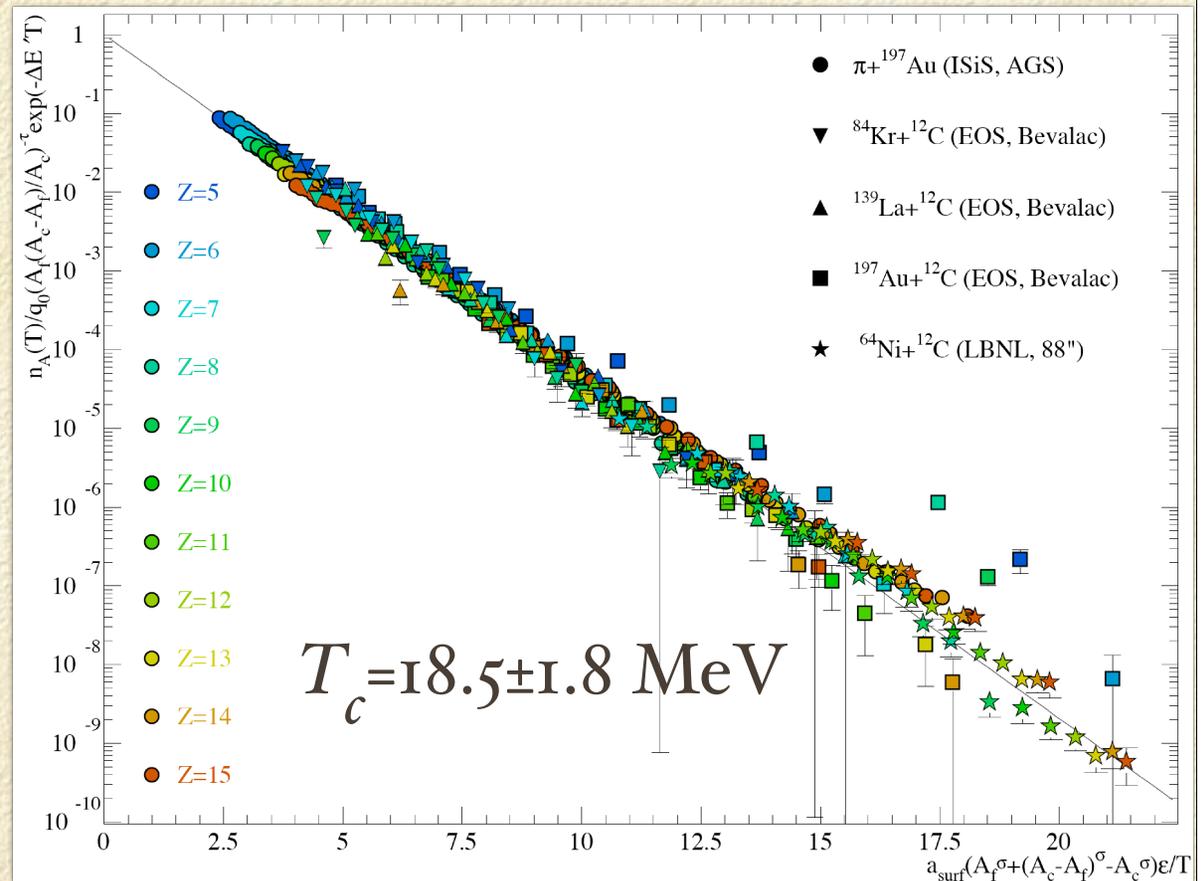
${}^{139}\text{La} + {}^{12}\text{C}$

${}^{197}\text{Au} + {}^{12}\text{C}$

$E/A = 1 \text{ GeV}$

${}^{64}\text{Ni} + {}^{12}\text{C}$

$E/A = 6\text{--}14 \text{ MeV}$



Thermodynamic aside

- Principle of corresponding states:

- Cubic coexistence curve.
- Empirically given by:

$$\frac{\rho_{l,g}}{\rho_c} = 1 + \frac{3}{4} \left(1 - \frac{T}{T_c}\right) \pm \frac{7}{4} \left(1 - \frac{T}{T_c}\right)^{1/3}$$

- + for liquid
- - for vapor.

- Observed empirically in many fluids:

E. A. Guggenheim, *J. Chem. Phys.* **13**, 253 (1945).

J. Verschaffelt, *Comm. Leiden* **28**, (1896).

J. Verschaffelt, *Proc. Kon. Akad. Sci. Amsterdam* **2**, 588 (1900).

D. A. Goldhammer, *Z.f. Physike. Chemie* **71**, 577 (1910).

- 1/3 is critical exponent $\beta \approx 0.328$

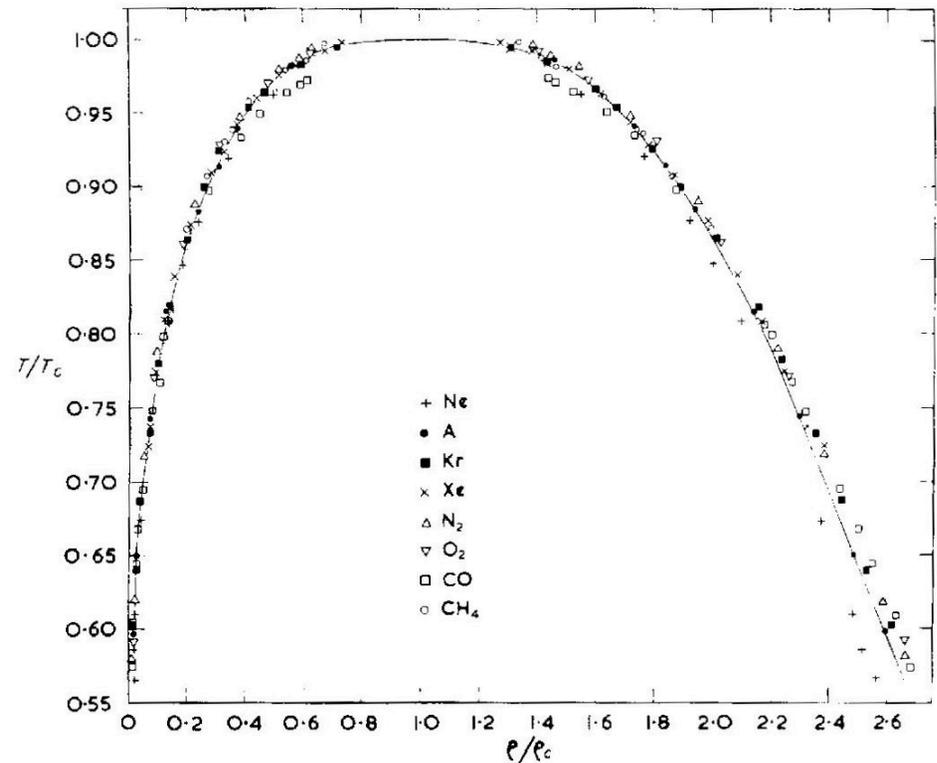


Fig. 3.11. Reduced densities of coexisting liquid and gas phases

Density phase diagram for infinite system

- Density (vapor branch)

$$\rho = q_0 \sum_A A^{1-\tau} \exp\left(-c_0 A^\sigma \left(\frac{1}{T} - \frac{1}{T_c}\right)\right)$$

- Liquid branch, Guggenheim scaling

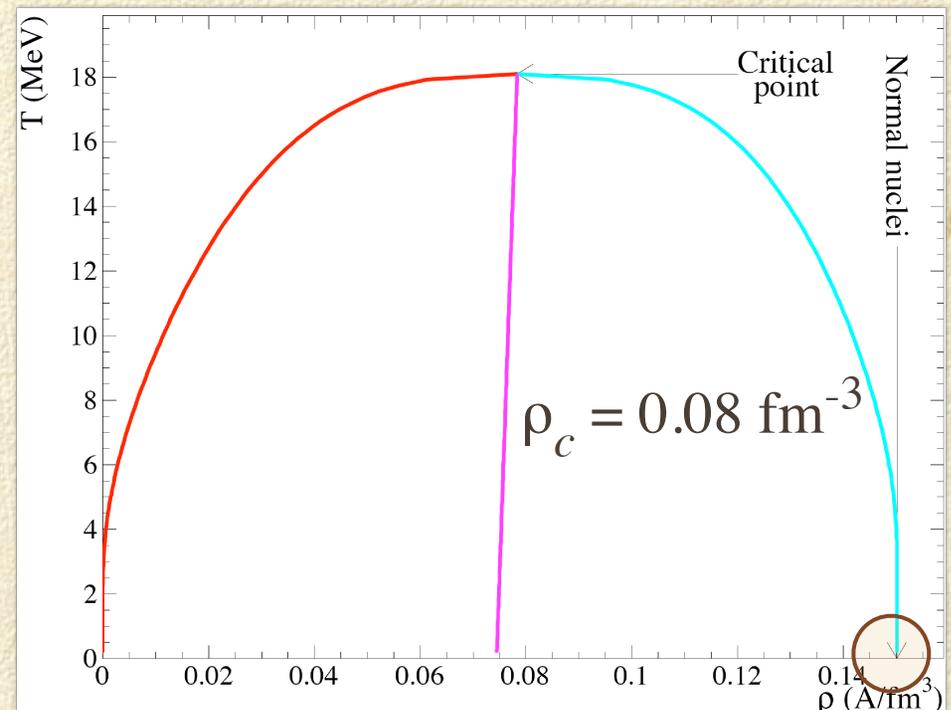
$$\frac{\rho}{\rho_c} = 1 + b_1 \epsilon + b_2 \epsilon^\beta$$

- b_1 and b_2 are fit parameters

$$\epsilon = \frac{T_c - T}{T_c} \quad \beta = \frac{\tau - 2}{\sigma} = .327$$

- Change sign of b_1 for the liquid density

$$\frac{\rho_l}{\rho_c} = 1 - b_1 \epsilon + b_2 \epsilon^\beta$$



$T=0$ is normal density; this sets the absolute density scale

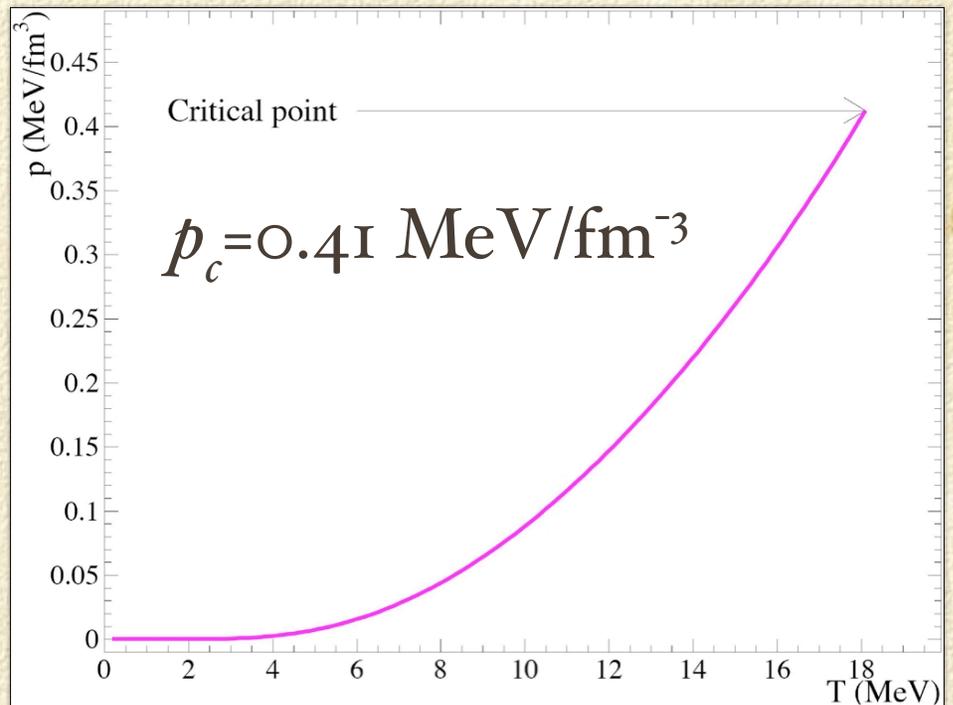
Pressure phase diagram for infinite system

□ Pressure

$$p = q_0 T \sum_A A^{-\tau} \exp\left(-c_0 A^\sigma \left(\frac{1}{T} - \frac{1}{T_c}\right)\right)$$

□ At T_c

$$p_c = q_0 T_c \sum_A A^{-\tau} = q_0 T_c \zeta(\tau)$$



Conclusions

- Solved three problems in the way of building a phase diagram of nuclear matter
 1. Finite size effects: use complement
 2. Coulomb: use transition state
 3. No physical vapor: use rate picture
- Now have a determination of the liquid-vapor coexistence line for infinite nuclear matter

T_c (MeV)	ρ_c (fm ⁻³)	p_c (MeV/fm ³)
18.5±1.8	0.077±0.018	0.41±.18

Lessons learned: physical picture (clusters)

- T_c and c_0 appear **together**
- You will get T_c wrong if you don't understand your finite size effects.
- T_c is a property of the infinite system. Didn't observe it. Couldn't even reach it. But all data point to it.
- We must carefully define the phases first before we discuss phase transitions. (Coulomb)
- 1st order phase transitions are trivial; they can be described in terms of the thermodynamic properties of each phase considered independently.

Lesson learned: discovery

- Clausius-Clapeyron Equation:

$$p = p_0 \exp\left(\frac{-\Delta H}{T}\right), \text{ valid when:}$$

- vapor pressure \sim ideal gas
- molar enthalpy $H_{\text{evaporation}}$ independent of T

- Neither true as $T \sim T_c$:

– The two deviations compensate:

$$\frac{p}{p_c} = \exp\left[\frac{-\Delta H}{T_c} \left(1 - \frac{T_c}{T}\right)\right]$$

- Observed empirically for several fluids:

“Thermodynamics” E. A. Guggenheim.

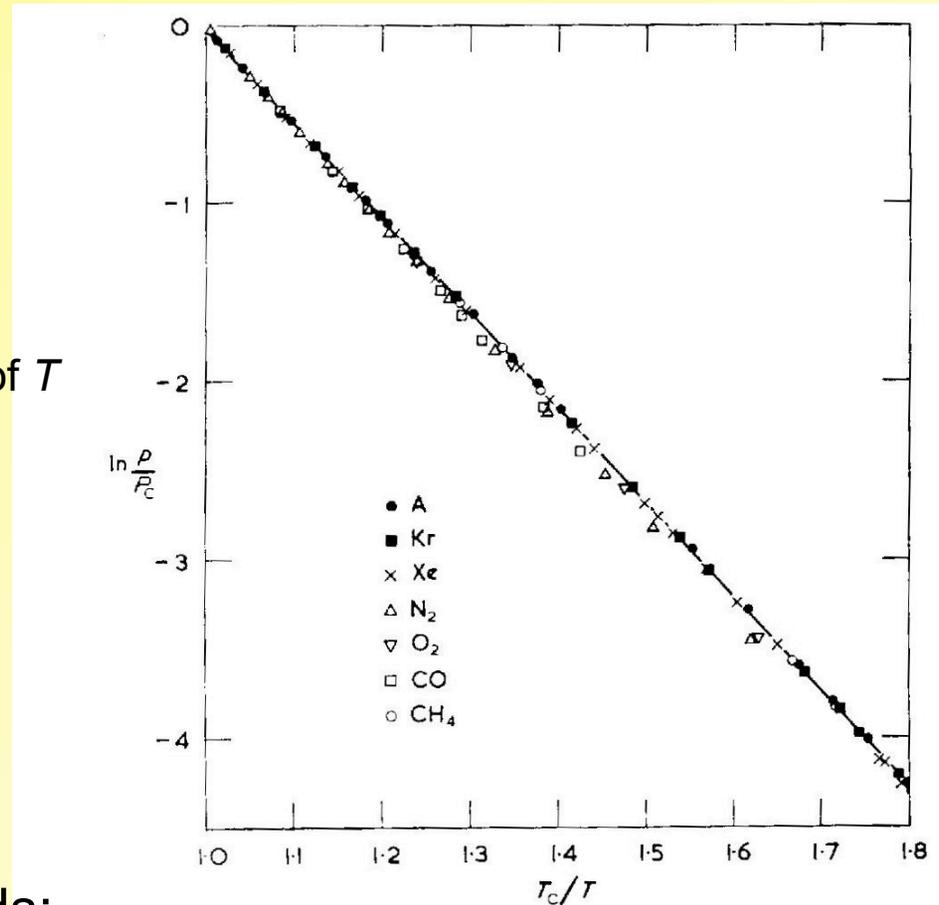


Fig. 3.12. Relation between vapour pressure and temperature

e^+e^- collisions?

Discovery occurred shortly after neutrons could be described as evaporating

Lessons learned: what did not work

- Anything that had to do with fluctuations
 - Intermittency, moments of the mass distributions, etc. – finite size effects
- Energy scans, vary system size, etc.
 - Finite size effects prevent us from ever reaching the critical point.
 - However, with the right physical picture, all data “point” to the critical temperature

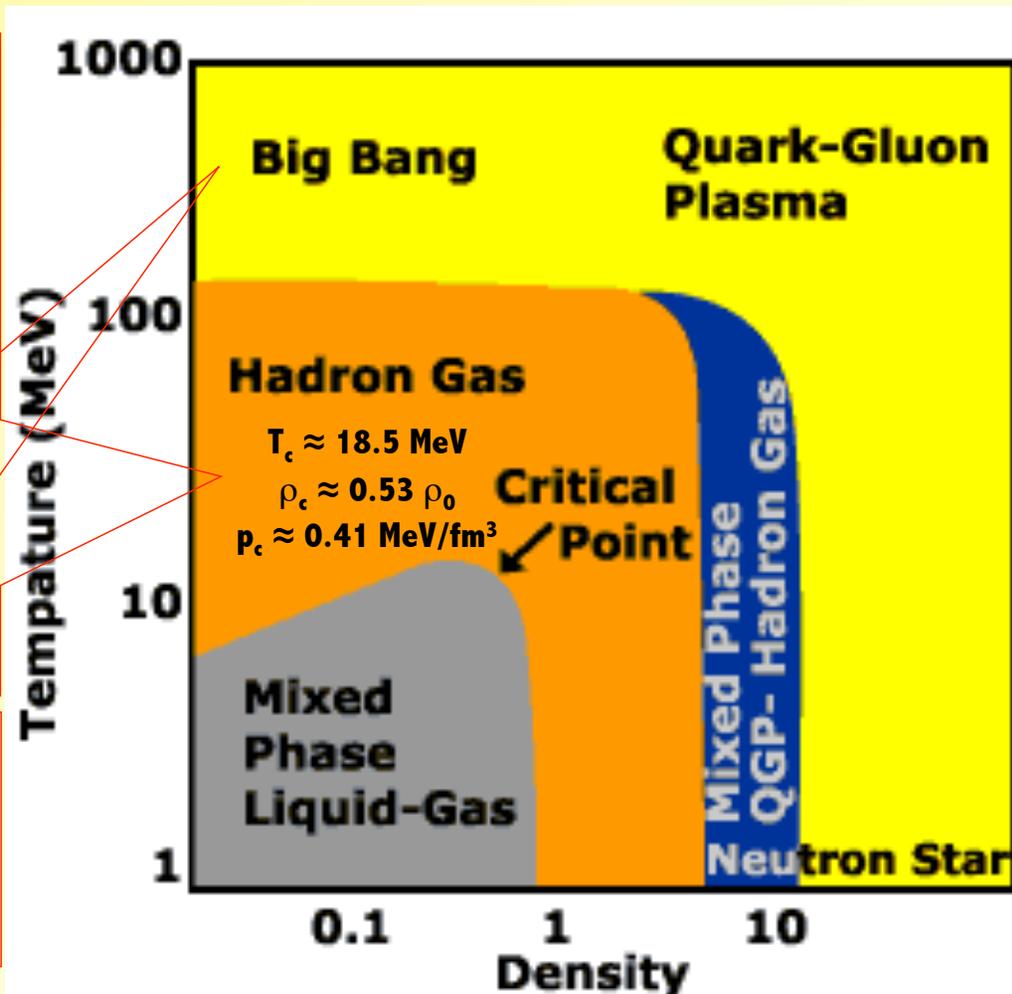
Phase transitions from Hadronic to Partonic Worlds

Phase transitions in the Hadronic world

- Pairing (superconductive) Transition
finite size effects: correlation length
- Shape transition
all finite size effects, shell effects
- Liquid-vapor (with reservations) van der Waals-like
finite size effects due to surface

Phase transitions in the partonic world

- QGP
Finite size effects?



Using the lessons learned

	Real fluids	Liquid-vapor	QGP
Order parameter?	density	density	?
phases defined?	yes	yes	?
Finite size effects?	not relevant	yes, but solved	?
T_c and another observable?	yes, surface energy	yes, surface energy	?
Physical picture	ideal gas law	ideal gas law	?
First order phase transition	yes	yes	?
second (higher order) phase transition	yes at T_c and ρ_c	yes, at T_c and ρ_c for symmetric neutral infinite nuclear matter	?

Thank you

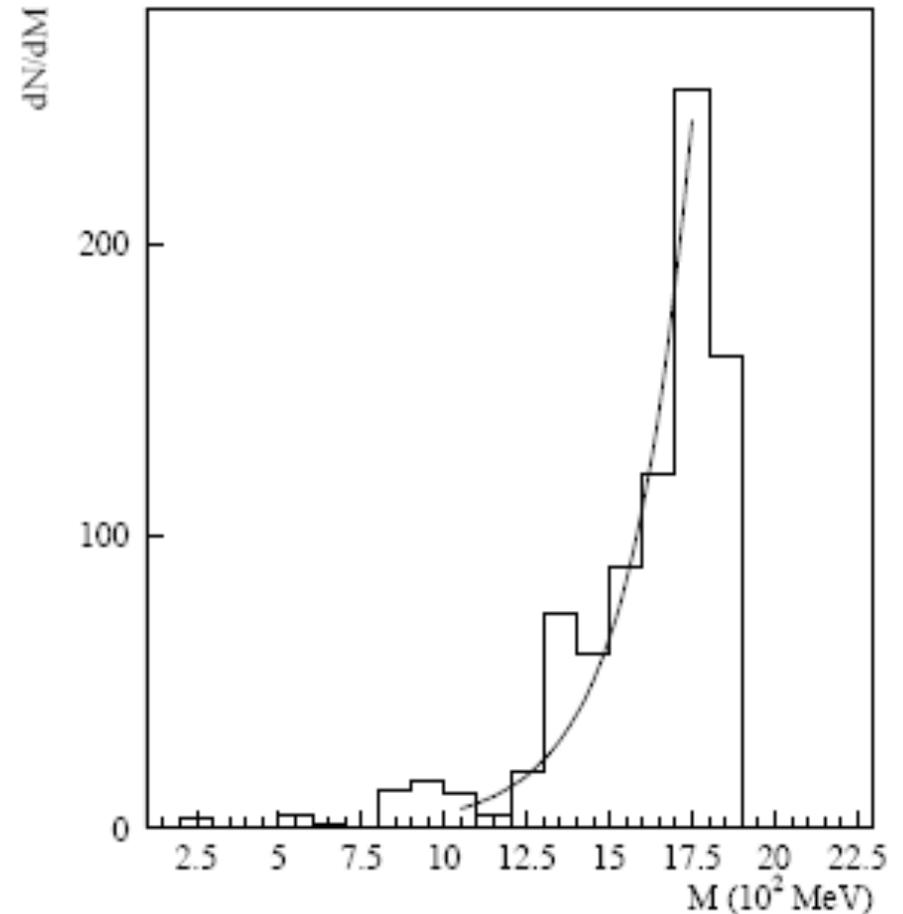
- A gas of bags : the gas that wasn't.
Instability against coalescence
- Criticality?
Fractality of bags.
- Can surface energy cure anything?
NO

The source of all troubles.....

The Hagedorn spectrum !

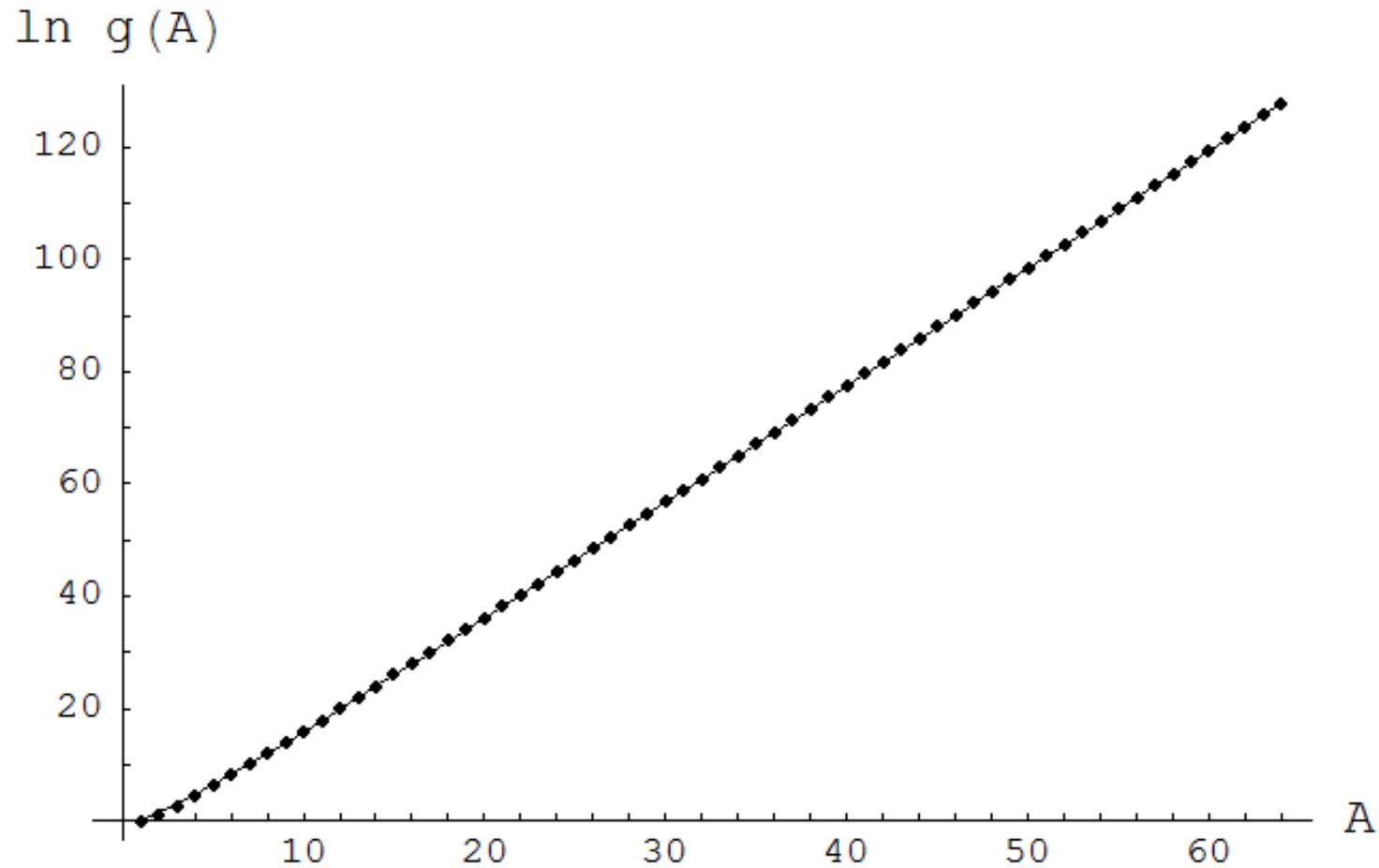
The (too) many ways of obtaining the Hagedorn spectrum (given the experimental evidence!!)

1. Bootstrap
2. MIT Bag Model
3. Regge Trajectories
4. Fractal shapes (if no surface energy)
5. -----



$$g(A) = 0.120705 A^{-1.38675} \exp(2.11982 A)$$

**Total Degeneracy of Lattice
Animals of a Given Size**



The partonic world (QGP) (a world without surface?)

- The MIT bag model says the pressure of a QGP bag is constant:

- $p = \frac{g\pi^2}{90} T_H^4 = B$; g : # degrees of freedom, constant $p = B$, constant

$$T_H = \left(B \frac{90}{g\pi^2} \right)^{\frac{1}{4}}$$

- The enthalpy density is then

- $\varepsilon = \frac{H}{V} = \frac{E}{V} + p = \frac{g\pi^2}{30} T_H^4 + B$

- which leads to an entropy of

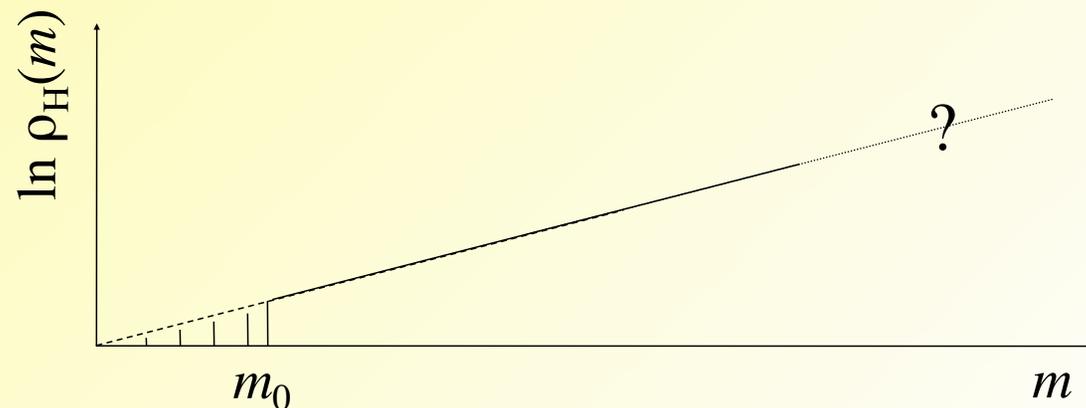
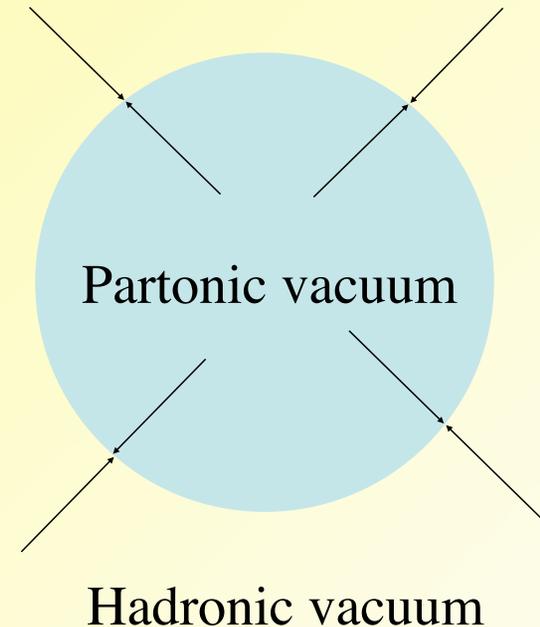
- $S = \int \frac{\delta Q}{T} = \int_0^H \frac{dH}{T} = \frac{H}{T_H} \equiv \frac{m}{T_H}$

- and a bag mass/energy spectrum (level density) of

$$\rho(m) = \exp(S) \propto \exp\left(\frac{m}{T_H}\right)$$

- This is a Hagedorn spectrum:

- $\rho_H(m) \propto \left(\frac{m_0}{m}\right)^x \exp\left(\frac{m}{T_H}\right)$



Can a “thermostat” have a temperature other than its own?



$$T = T_c = 273\text{K} \quad ?$$

or

$$0 \leq T \leq 273\text{K} \quad \bullet$$

- $S = S_0 + \frac{\Delta Q}{T} = S_0 + \frac{E}{T_0}$
- $\rho(E) = e^S = e^{S_0 + \frac{E}{T_0}}$
- Is T_0 just a “parameter”?
- $Z(T) = \int dE \rho(E) e^{-E/T} = \frac{T_0 T}{T_0 - T} e^{S_0}$
- According to this, a thermostat, can have any temperature lower than its own!

Equilibrium with Hagedorn bags:

Example : an ideal vapor of N particles of mass m and energy ε

- The total level density:

$$P(E, \varepsilon) = \rho_H(E - \varepsilon) \rho_{iv}(\varepsilon) = g(m) \frac{V^N}{N! \left(\frac{3}{2}N\right)!} \left(\frac{m\varepsilon}{2\pi}\right)^{\frac{3}{2}N} \exp\left(\frac{E - mN - \varepsilon}{T_H}\right)$$

- Most probable energy partition:

$$\frac{\partial \ln P}{\partial \varepsilon} = \frac{3N}{2\varepsilon} - \frac{1}{T_H} = 0 \Rightarrow \frac{\varepsilon}{N} = \frac{3}{2} T_H$$

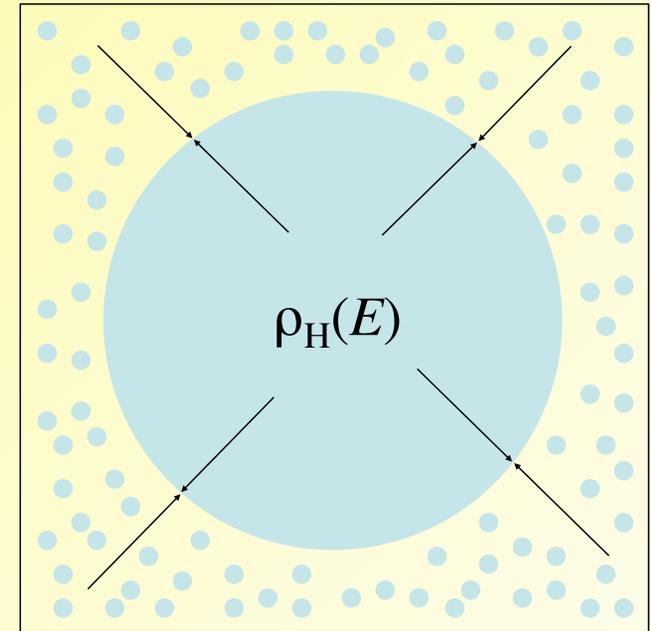
- T_H is the **sole temperature** characterizing the system:

- A Hagedorn-like system is a perfect thermostat.

- If particles are generated by the Hagedorn bag, their concentration is:

$$\left. \frac{\partial \ln P}{\partial N} \right|_V = -\frac{m}{T_H} + \ln \left[g(m) \frac{V}{N} \left(\frac{mT_H}{2\pi}\right)^{\frac{3}{2}} \right] = 0 \Rightarrow \frac{N}{V} = g(m) \left(\frac{mT_H}{2\pi}\right)^{\frac{3}{2}} \exp\left(-\frac{m}{T_H}\right)$$

- **Volume independent! Saturation!** Just as for ordinary water, but with only one possible temperature, T_H !

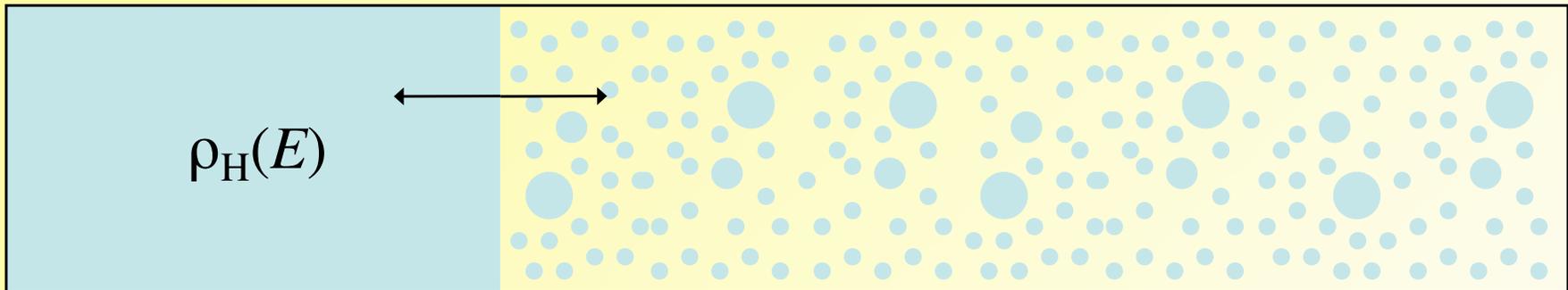


ideal vapor ρ_{iv}

- particle mass = m
- volume = V
- particle number = N
- energy = ε

The story so far . . .

1. Anything in contact with a Hagedorn bag acquires the temperature T_H of the Hagedorn bag.
2. If particles (e.g. π s) can be created from a Hagedorn bag, they will form a saturated vapor at fixed temperature T_H .



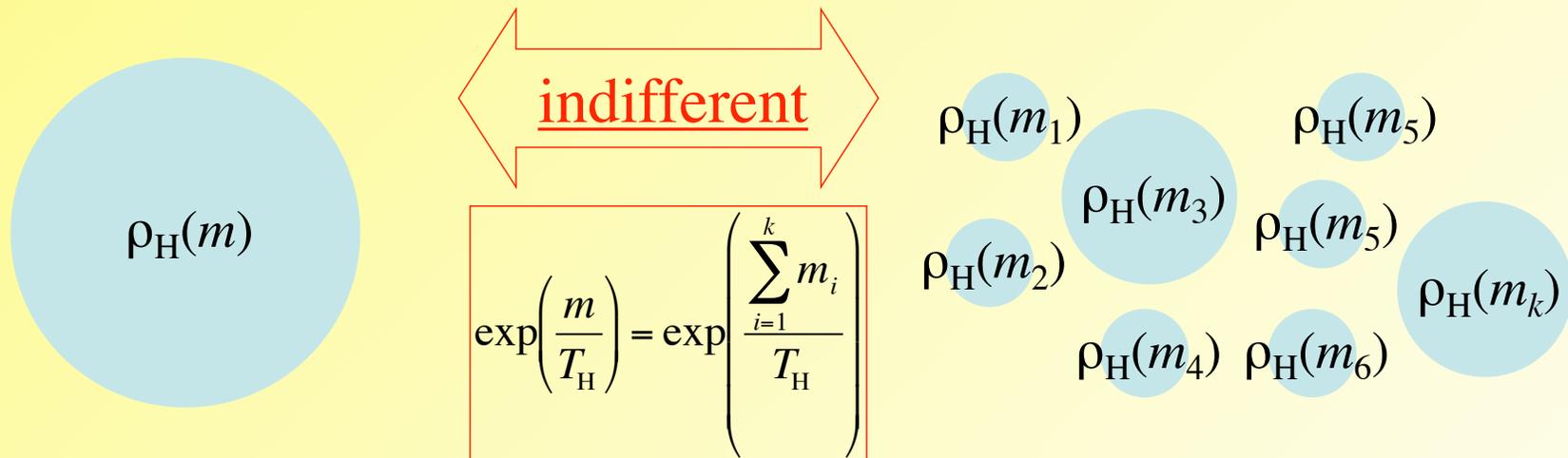
5. If different particles (i.e. particles of different mass m) are created they will be in chemical equilibrium.

Now to the gas of bags ...

(Gas of resonances?)

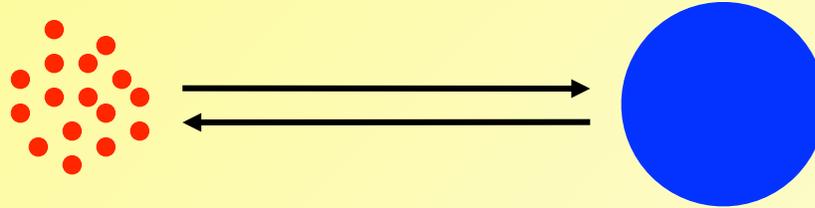
Stability of the Hagedorn bag against fragmentation

- If no translational or positional entropy, then the Hagedorn bag is indifferent to fragmentation.



Resonance gas - A gas without pressure

No intrinsic energy and/or entropy penalty for aggregation



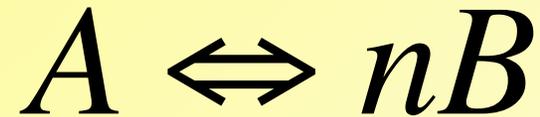
How many particles?

$$1 \leq N \leq N_{\max}$$

Ideal gas law:

$$p = \frac{N}{V} T$$

Resonance Gas Cont'd



$$\frac{c_B^n}{c_A} = \frac{q_B^n}{q_A} = \frac{q_A^{n-1}}{n^{3n/2}}$$

$$q_B = \left(\frac{2\pi m_A T}{h^2 n} \right)^{3/2} = \frac{1}{n^{3/2}} q_A$$

$$\lim_{n \rightarrow \infty} c_B = \lim_{n \rightarrow \infty} \frac{c_A^{1/n} q_A^{n-1}}{n^{3n/2}} = 0$$

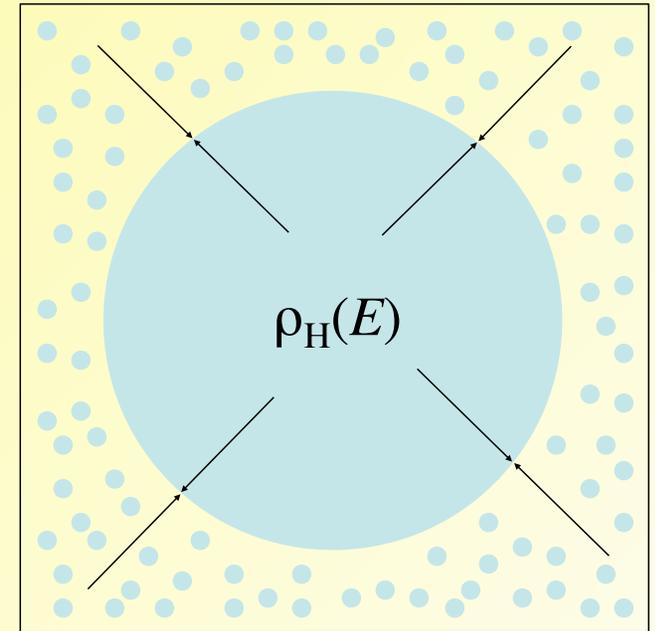
$$\% \text{dissoc} = \frac{1}{n^{5/2}} q_A^{n-1} \frac{c_A^{1/n}}{c_0}$$

Equilibrium with Hagedorn bags:

$$\frac{N}{V} = g(m) \left(\frac{mT_H}{2\pi} \right)^{\frac{3}{2}} \exp\left(-\frac{m}{T_H} \right)$$

$$g(m) = e^{\frac{m}{T_H}}$$

$$\frac{N}{V} = \left(\frac{mT_H}{2\pi} \right)^{\frac{3}{2}}$$



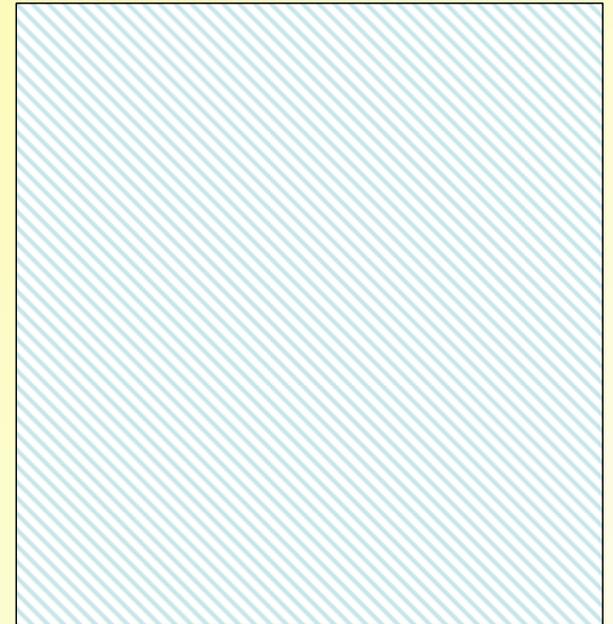
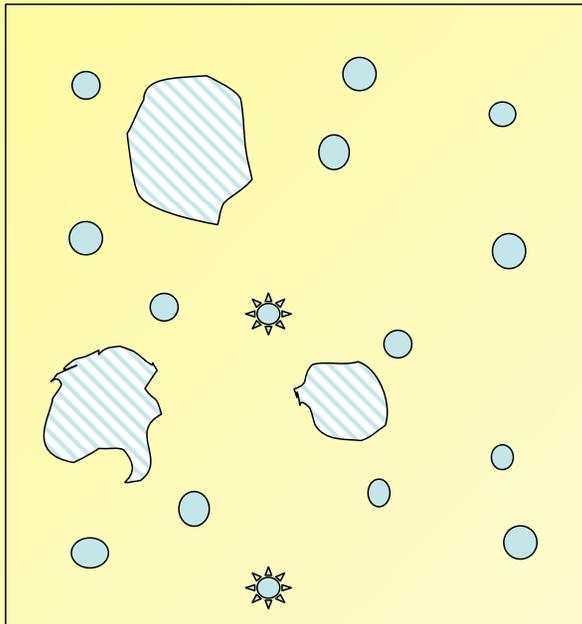
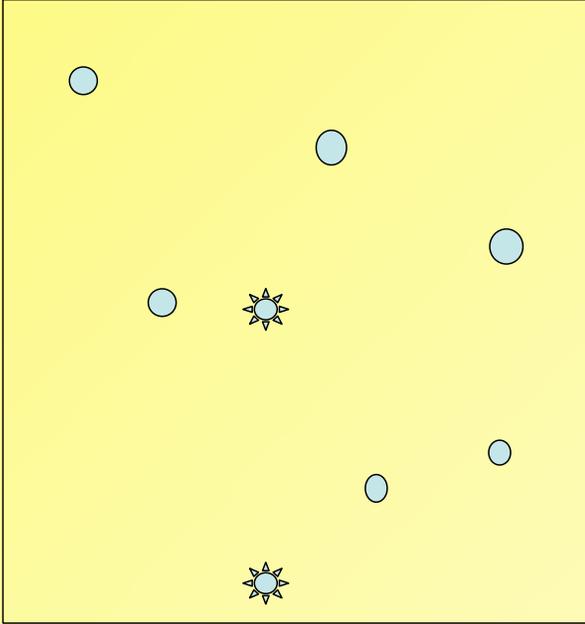
ideal vapor ρ_{iv}

- particle mass = m
- volume = V
- particle number = N
- energy = ϵ

$T < T_H$

$T = T_H$

$T = T_H$

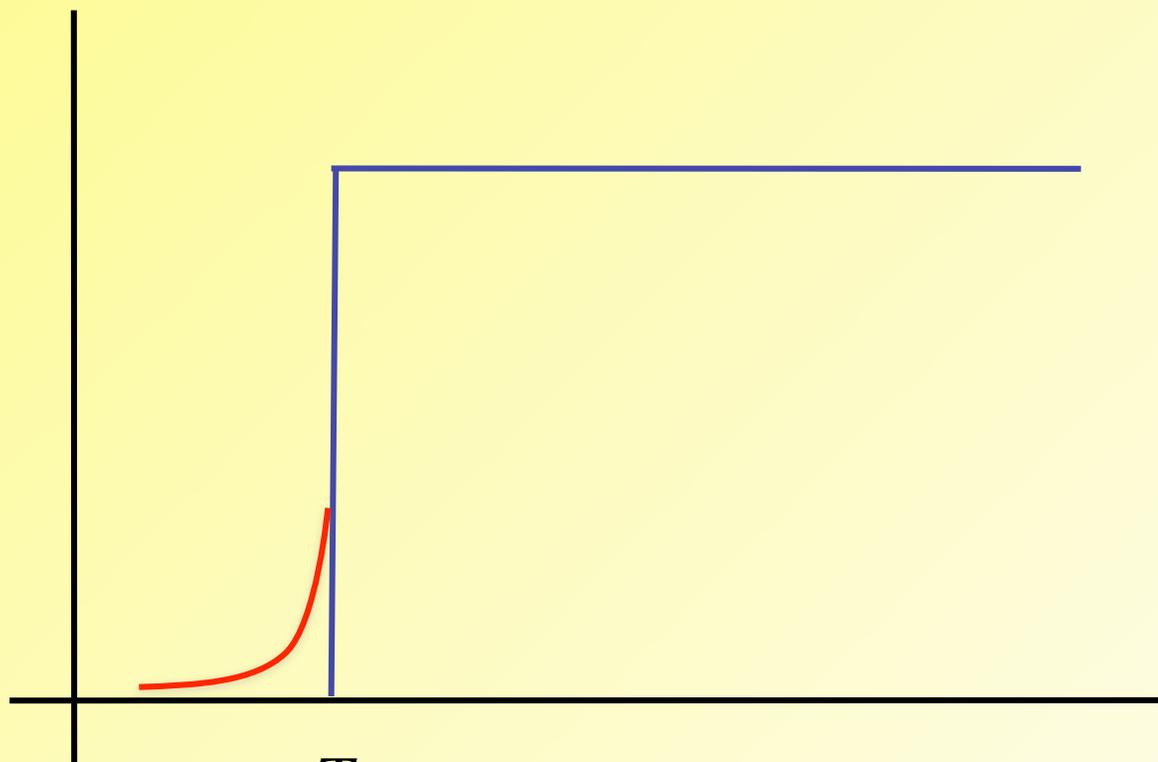


Non saturated gas of π
etc.

Gas of bags +
saturated gas of π etc.

One big bag

$$\frac{\varepsilon}{T^4}$$

 T_H T

Bags have no surface energy :

What about criticality?

Fisher Criticality

$$P(A) = K g(A) e^{-\frac{C_s A^{2/3}}{T}} = K A^{-\tau} e^{\frac{C_s A^{2/3}}{T_{cr}}} e^{-\frac{C_s A^{2/3}}{T}}$$

$$? \quad g(A) \cong A^{-\tau} e^{KA^{2/3}} \quad ?$$

$$\ln g(A) = \text{Surface Entropy} = KA^{2/3} = \frac{C_s}{T_{cr}} A^{2/3}$$

This is predicated upon a nearly spherical cluster.

? ! True ?!

Lattice Animals

			///		
		///	///		
			///	///	
	///				
	///				
///					

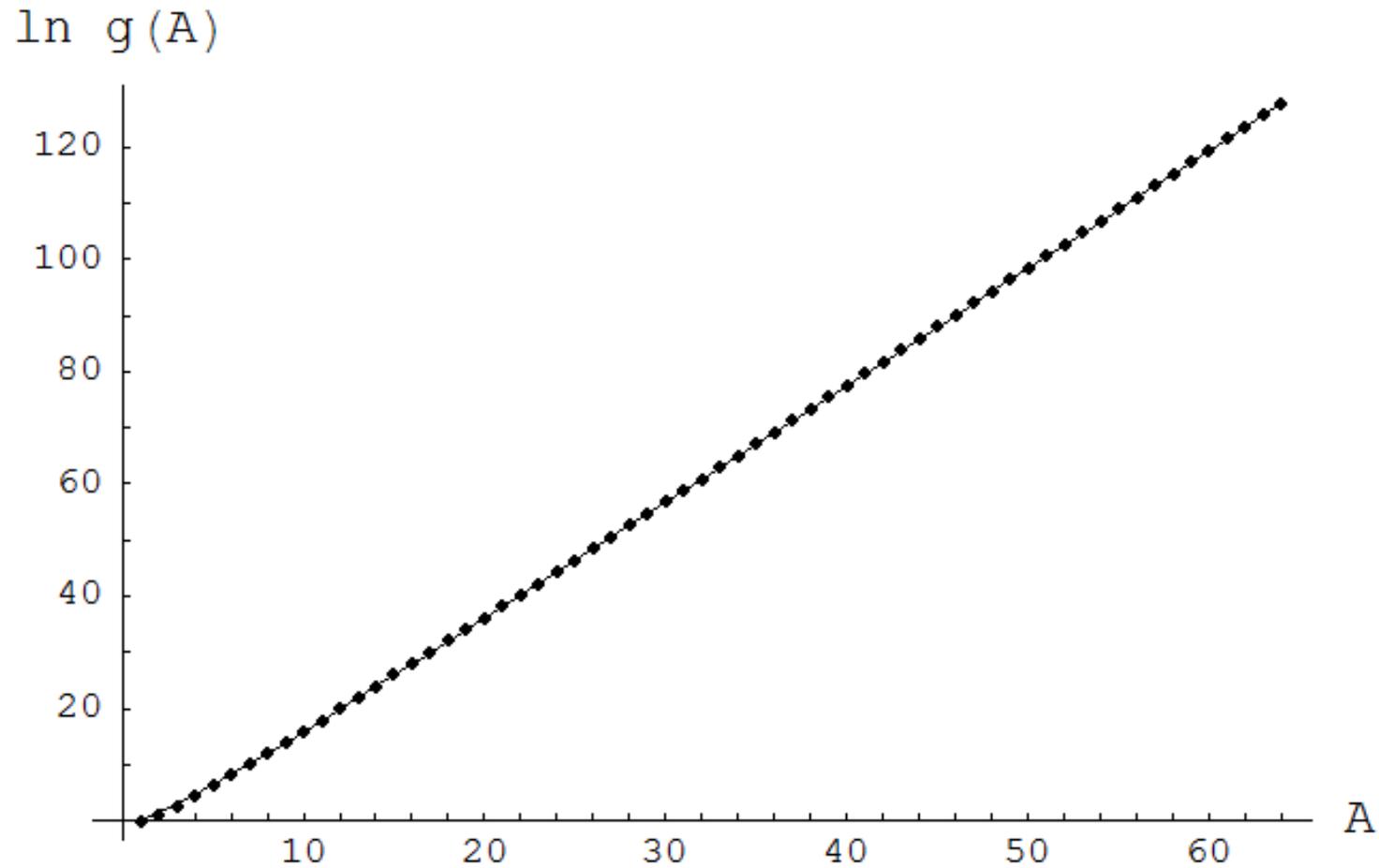
How many animals of size A ?

Fisher guesses $\ln P(A) = -\tau \ln A + KA^{2/3}$

To my knowledge nobody knows exactly why .

$$g(A) = 0.120705 A^{-1.38675} \exp(2.11982 A)$$

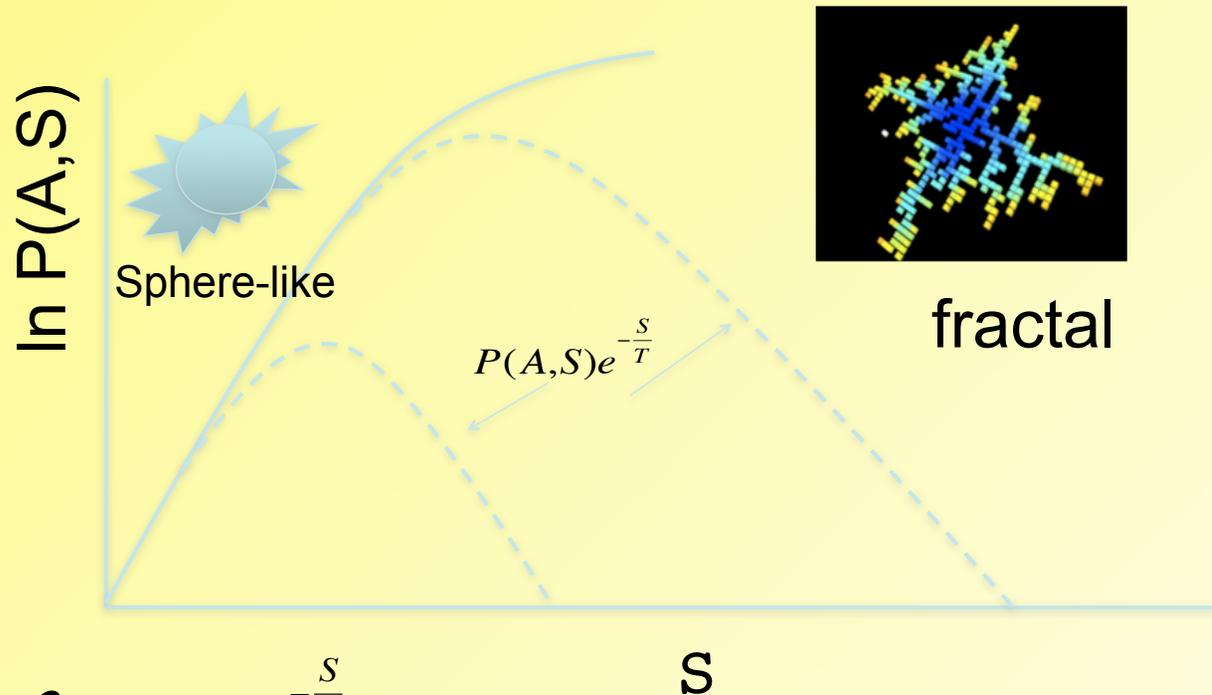
**Total Degeneracy of Lattice
Animals of a Given Size**



..... Instead

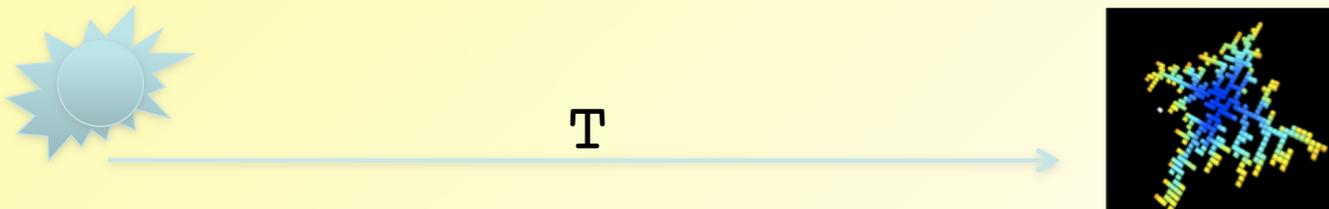
$$\ln P(A) = \alpha A^{2/3} + \beta A$$

How to resolve this conundrum?....



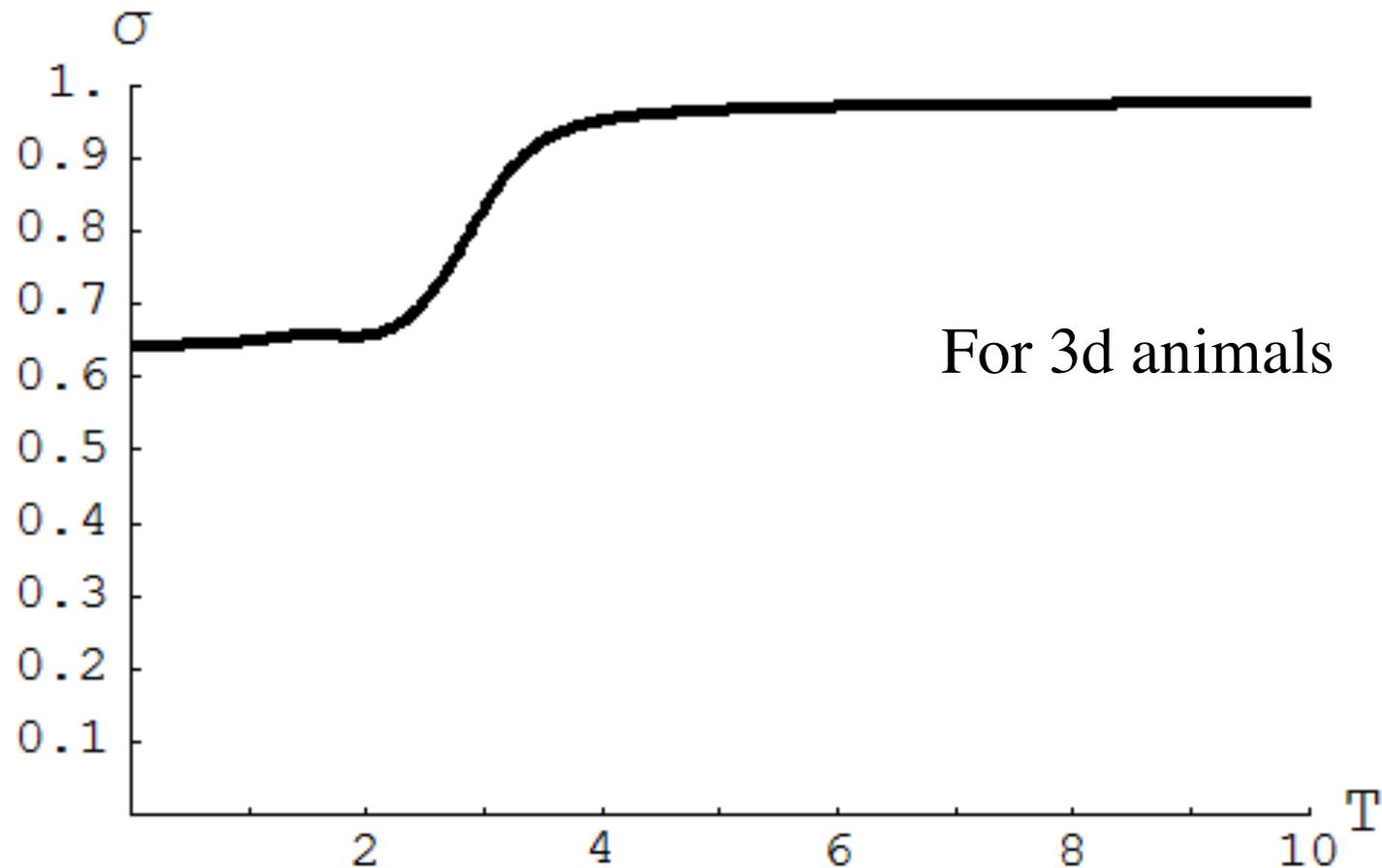
$$P(A,T) = \int P(A,S) e^{-\frac{S}{T}} ds$$

With increasing temperature ..



Fractal dimension goes from surface-like to volume-like

Surface Changes ($S \propto A^\sigma$) as a Function of Temperature



Back to the bags.....

No surface energy, no Boltzmann factor to keep the bag sphere-like.

So, at T_H , the only natural temperature of the bag
fractality \cong criticality
already appears,

although the coexistence of the bag with a non Hagedorn vapor is 1st order.

Can Surface Energy

Save the day?

A bag with a surface?

• Remember the leptodermous expansion:

$$\bullet M = E \cong H = a_V A + a_S A^{2/3} + a_C A^{1/3}$$

• Notice that in most liquids $a_S \approx -a_V$

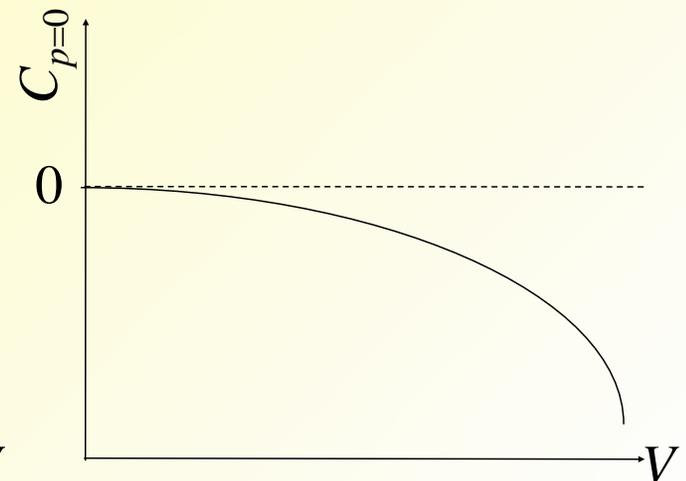
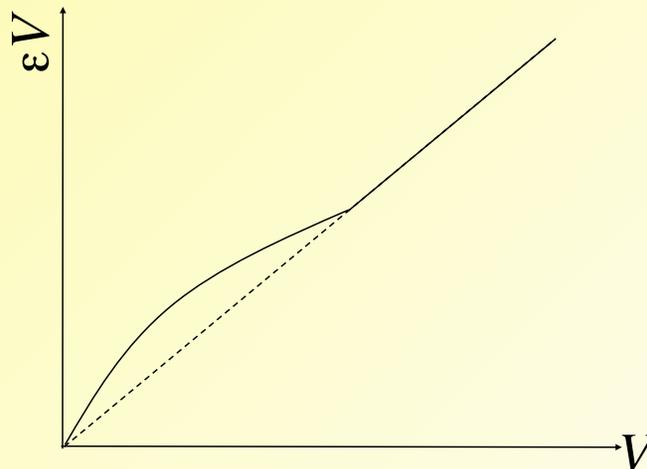
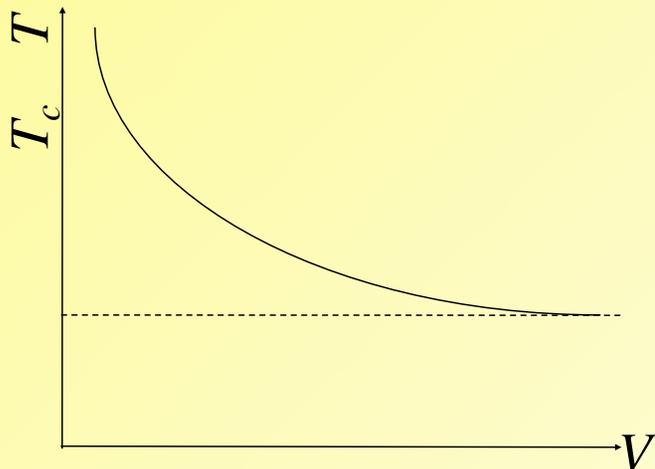
• However, in the MIT bag there is only a volume term

$$\bullet \varepsilon V = H = [f(T) + B]V + a_S V^{2/3} \quad (?)$$

• Should we introduce a surface term? Although we may not know the magnitude of it, we know the sign (+). The consequences of a surface term:

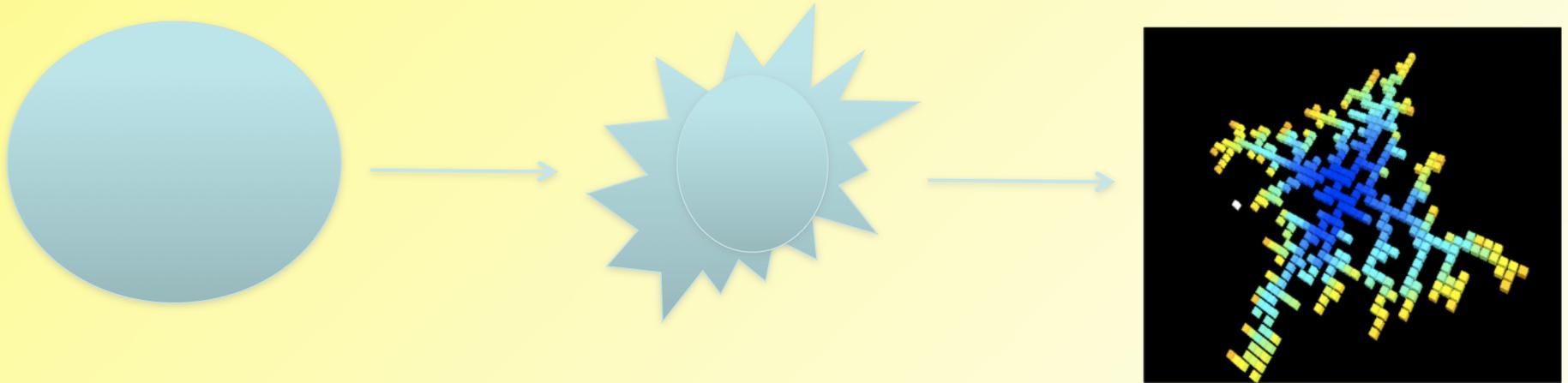
$$\bullet p = \frac{1}{3} f(T) - \left(B + \frac{2}{3} a_S V^{-1/3} \right) = 0 \quad \text{at equilibrium}$$

$$\bullet T = f^{-1} \left[3 \left(B + \frac{2}{3} a_S V^{-1/3} \right) \right]$$



Rumors have it that the bags surface energy coefficient may be negative ! ? !

$$C_s < 0$$

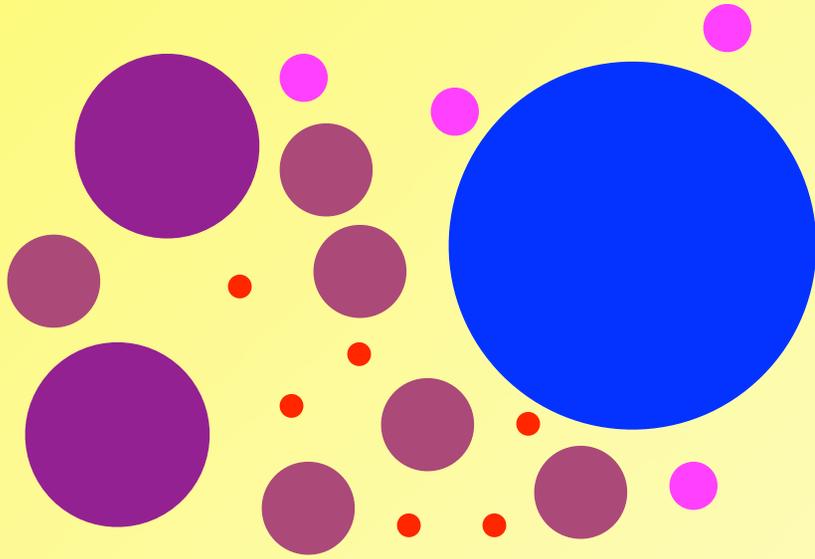


Could it be that :

What one throws away by neglecting surface entropy

One is forced (by data?) to put back with a negative C_s

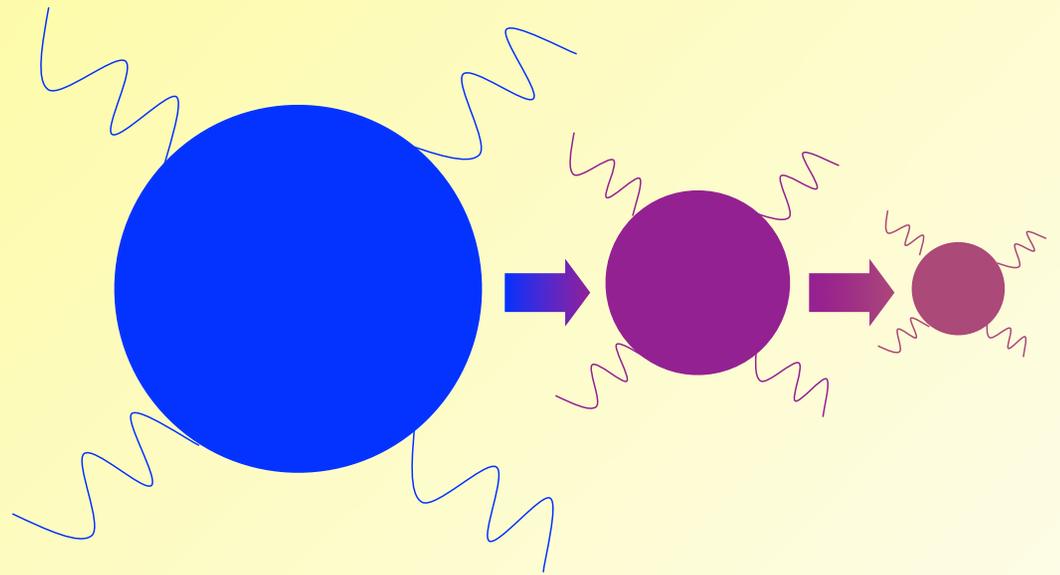
Stability of a gas of bags



Bags of different size are of different temperature. If the bags can fuse or fission, the lowest temperature solution at constant energy is a single bag. The isothermal solution of many equal bags is clearly unstable.

A gas of bags is always thermodynamically unstable.

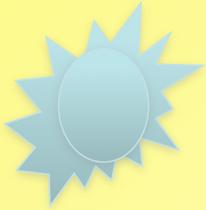
The decay of a bag with surface



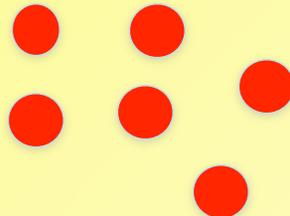
A bag decays in vacuum by radiating (e.g. pions). As the bag gets smaller, it becomes **HOTTER!** Like a mini-black hole.

Conclusions

1) The bag supports a 1st order phase transition



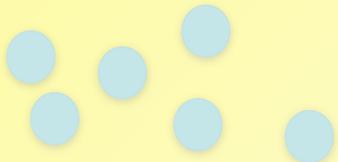
Bag



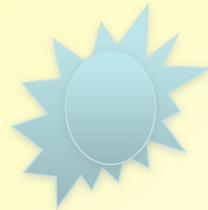
Non Hagedorn particles (pions?)

at a single T_H

2) A gas of bags is entropically unstable towards coalescence



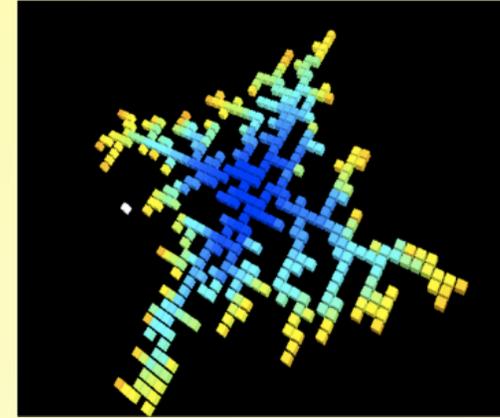
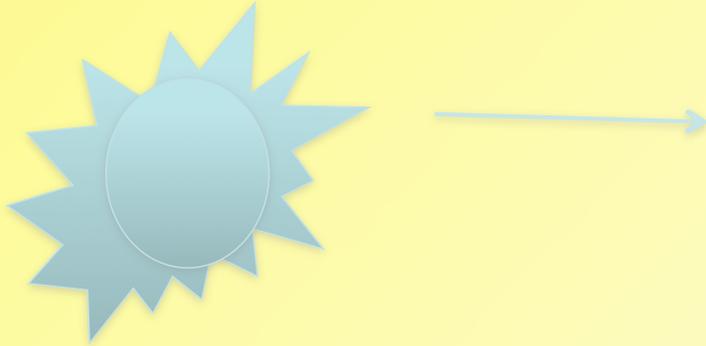
Hagedorn drops



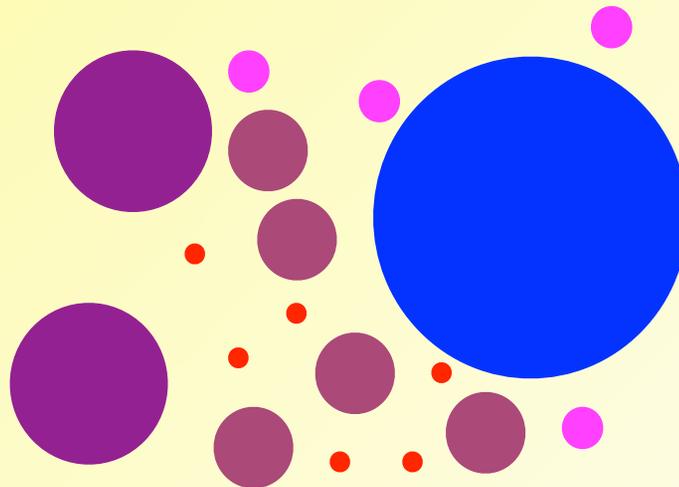
Bag

Conclusions ctd..

- 3) The lack of surface energy entropically drives bag to fractal shape

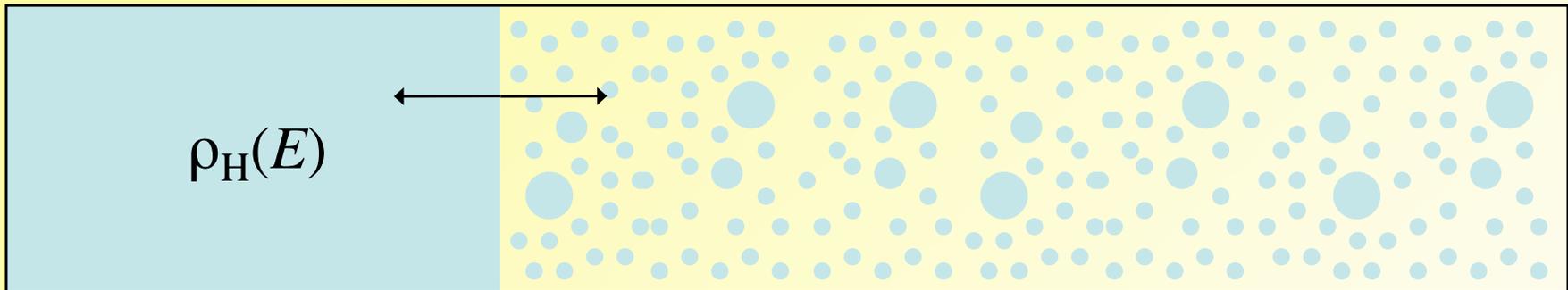


- 4) Addition of surface energy makes drops non isothermal.



The story so far . . .

1. Anything in contact with a Hagedorn bag acquires the temperature T_H of the Hagedorn bag.
2. If particles (e.g. π s) can be created from a Hagedorn bag, they will form a saturated vapor at fixed temperature T_H .



5. If different particles (i.e. particles of different mass m) are created they will be in chemical equilibrium.